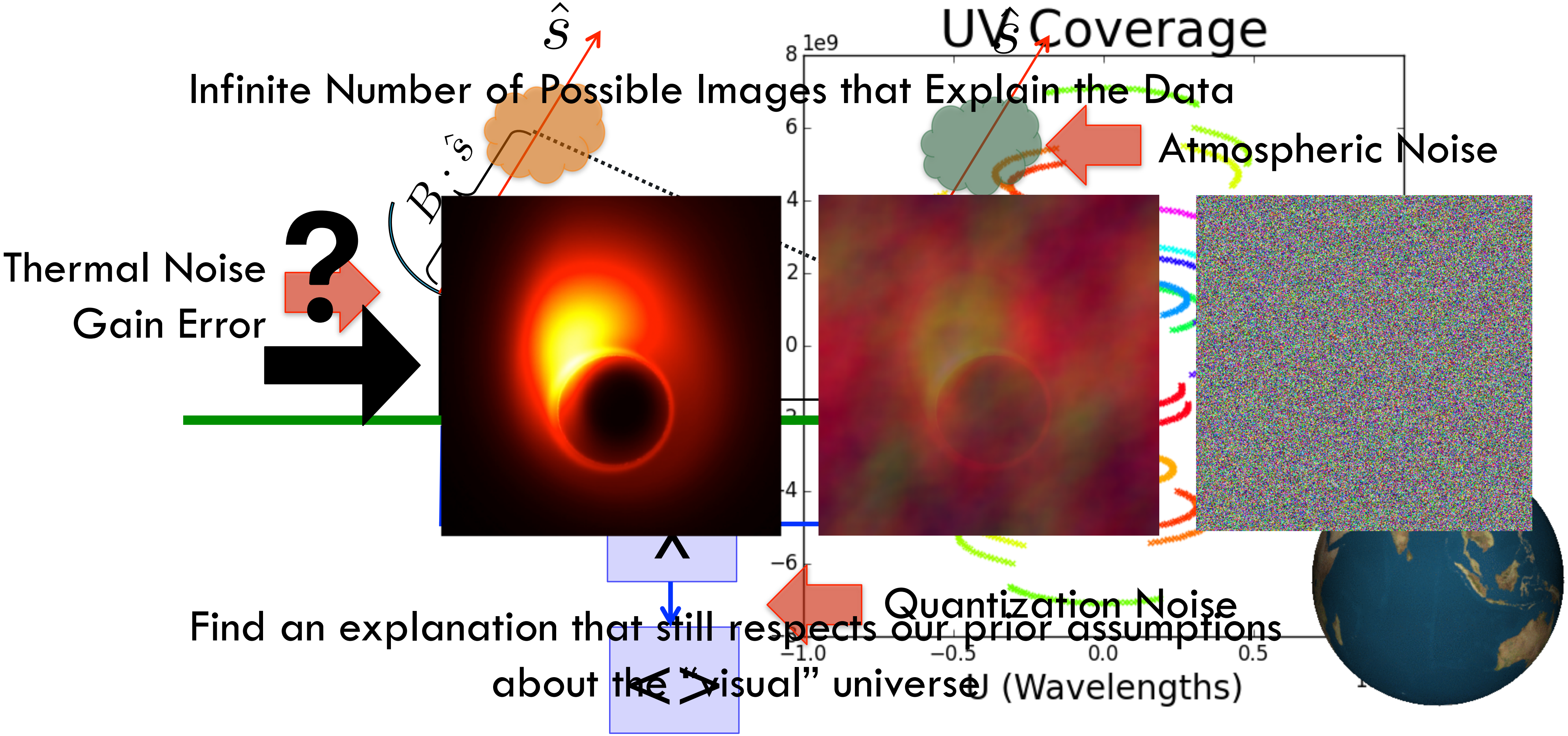
A Bayesian Algorithm & Dataset for mm-VLBI Image Reconstruction

Katie Bouman

Challenge of Image Reconstruction

Infinite Number of Possible Images that Explain the Data



Find an explanation that still respects our prior assumptions about the “visual” universe

Bayesian Inference

Bayes Law

$$P(I_\lambda|D) = \frac{P(D|I_\lambda)P(I_\lambda)}{P(D)}$$

Constant → $P(D)$

Best Image that Explains the Measurements

$$\hat{I}_{\lambda \mathrm{MAP}} = \mathrm{argmax}_{I_\lambda \in \Omega} P(I_\lambda|D) P(I_\lambda)$$

Probability of Image Given Measurements

Probability of Data Given Image

Likelihood

Probability of Image

Prior

Related Work

CLEAN

- Not Bayesian
- Difficult to Adapt

Optical Interferometry

- Bispectrum-MEM
- SQUEEZE

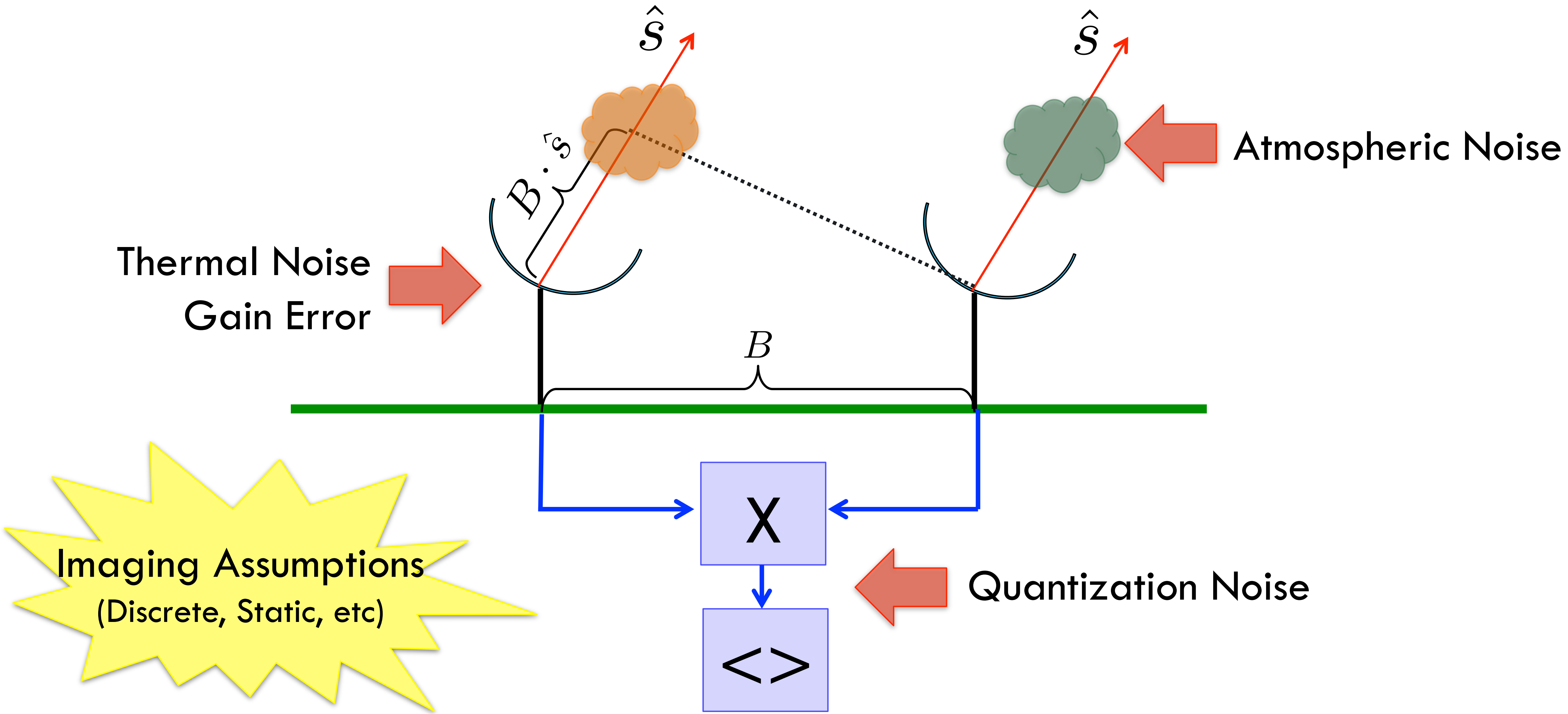
Overview

Image Reconstruction Algorithm

Likelihood
“Data Term”

Prior
“Previous Expectations Term”

Likelihood Term: Forward Modeling



Van Cittert-Zernike Theorem

Visibility

Time Averaged Cross Correlation

2D Fourier Transform of Continuous Image

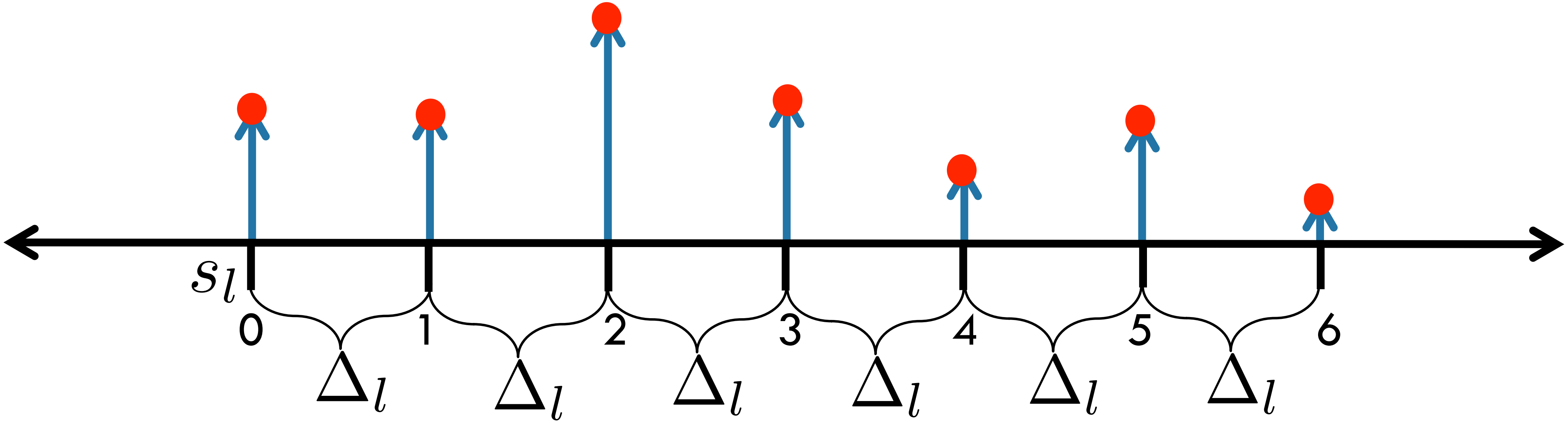
$$\Gamma(u,v) = \int_l \int_m e^{-i2\pi(ul+vm)} I_\lambda(l,m)\,dl\,dm$$

$$\approx \sum_{i=0}^{N_l-1} \sum_{j=0}^{N_m-1} e^{-i2\pi(u(\Delta_l i+s_l)+v(\Delta_m j+s_m))} X[i,j]$$

Discrete Space Fourier Transform : Direct Fourier Transform

Traditional Representation: Delta Pulse

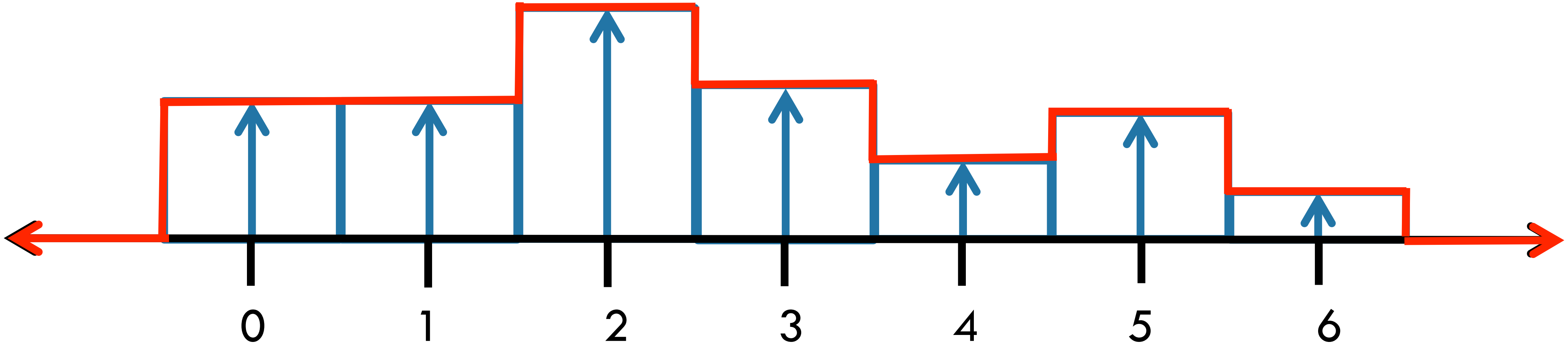
X[i] = 1.0 1.0 1.5 1.1 0.4 0.9 0.2



$$T(t) \approx \sum_{i=0}^{N_l-1} X[i]\,\delta\left(t - (\Delta_l i + s_l)\right)$$

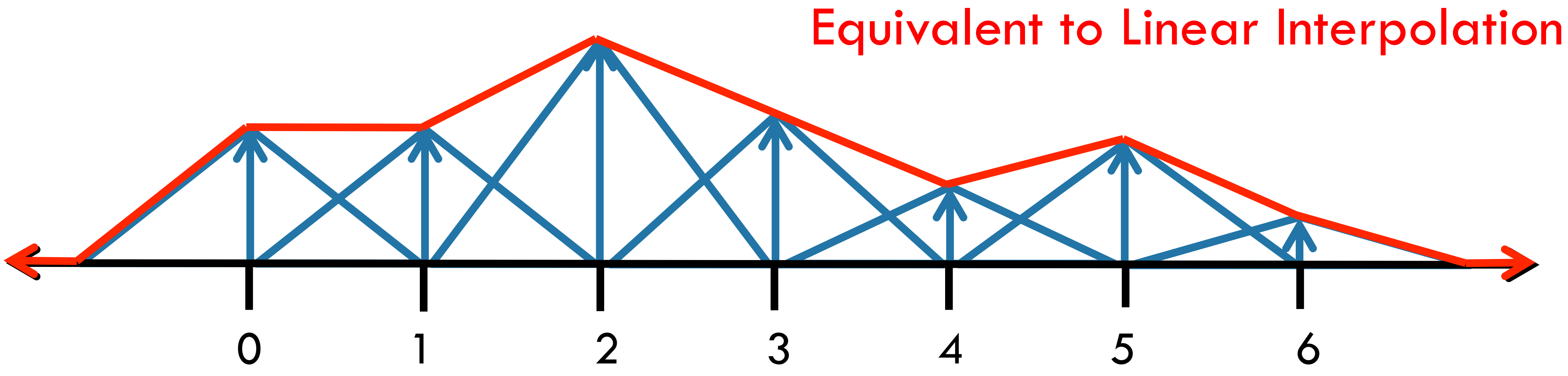
$$I_\lambda(u) \approx \sum_{i=0}^{N_l-1} X[i]\, e^{-i2\pi\left(u(\Delta_l i + s_l)\right)}$$

Image Representation: Rectangle Pulse



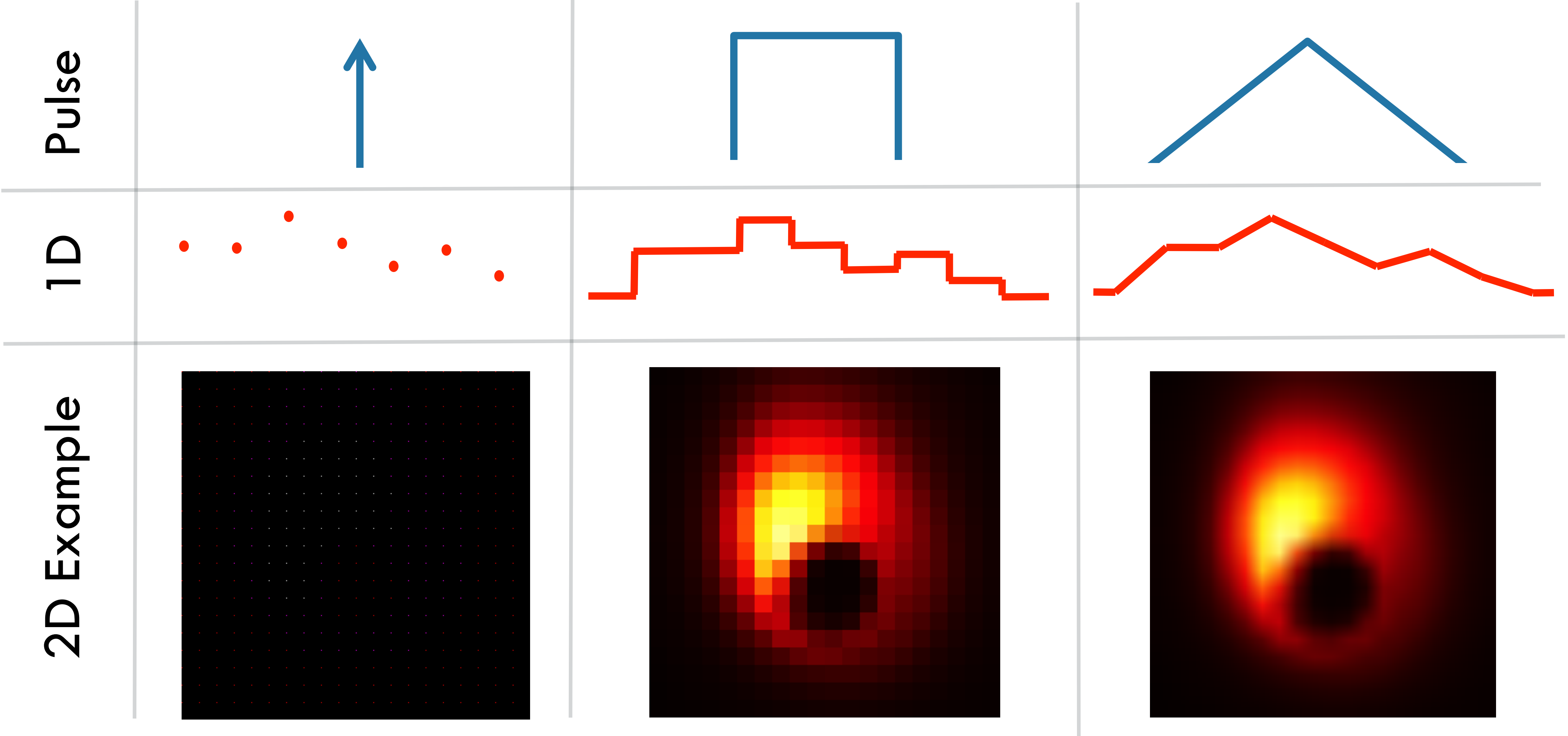
$$I_\lambda(l) \approx \sum_{i=0}^{N_l - 1} X[i] \sqcap (l - (\Delta_l i + s_l))$$

Image Representation: Triangle Pulse



$$I_\lambda(l) \approx \sum_{i=0}^{N_l - 1} X[i] \bigwedge (l - (\Delta_l i + s_l))$$

Comparing Image Pulses



Approximate Van Cittert-Zernike Theorem: 1D

$$\Gamma(u) = \int_l e^{-i2\pi ul} I_\lambda(l)\, dl$$

$$\Gamma(u) \approx \int_l e^{-i2\pi ul} \sum_{i=0}^{N_l-1} X[i]\, h\left(l - (\Delta i + s_l)\right)$$

$$e^{-i2\pi u(\Delta i + s_l)} H(u)$$

Fourier Transform of Shifted Pulse

Shift Theorem

Fourier Transform of Pulse

Approximate Van Cittert-Zernike Theorem: 2D

$$\Gamma(u,v) \approx \overbrace{H(u,v)}^{\text{Scalar}} \underbrace{\sum_{i=0}^{N_l-1} \sum_{j=0}^{N_m-1} e^{-i2\pi(u(\Delta_l i + s_l) + v(\Delta_m j + s_m))} X[i,j]}_{\text{Same Calculation as Before!}}$$

Works for Any Pulse With a Closed-Form Fourier Transform

Overview

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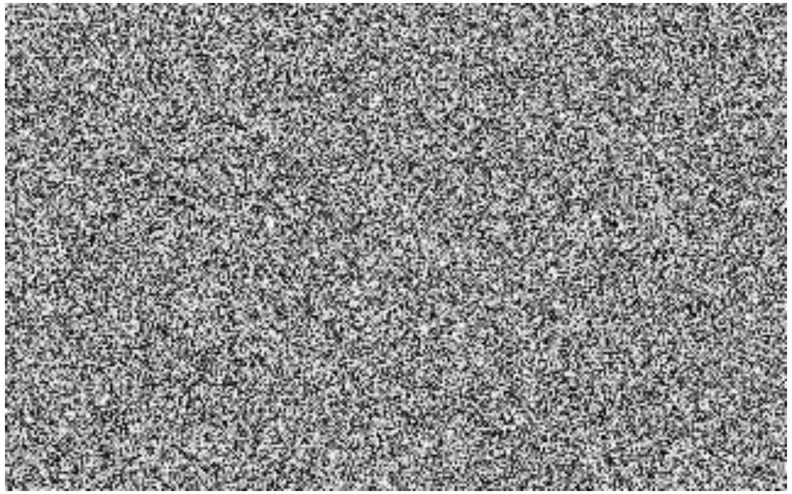
Some images adapted from slides by Daniel Zoran

Natural Image Prior

Given an NxN matrix X return P(X) - “Probability that X is a natural image”



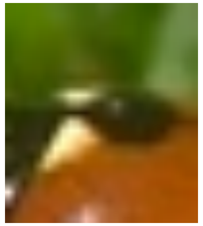
An unlikely image

A more likely image

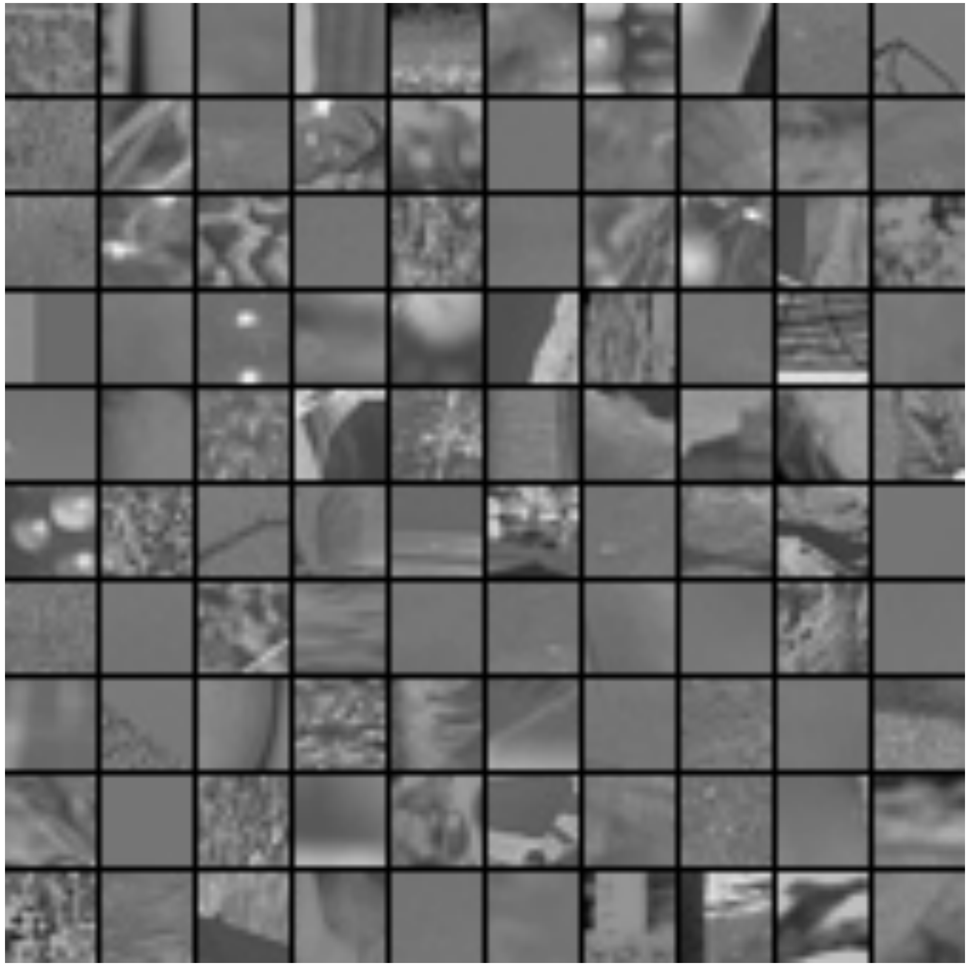
A likely image

Natural Patch Prior

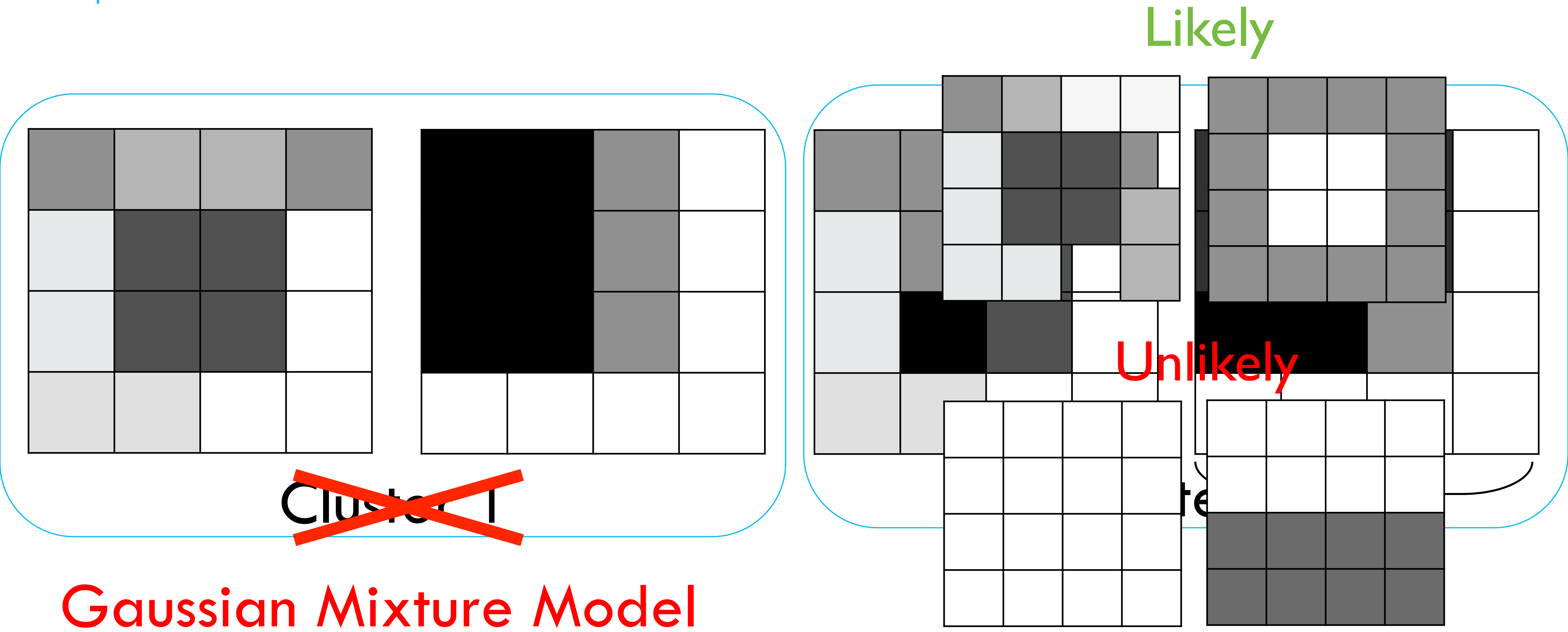
P(

) vs. P()

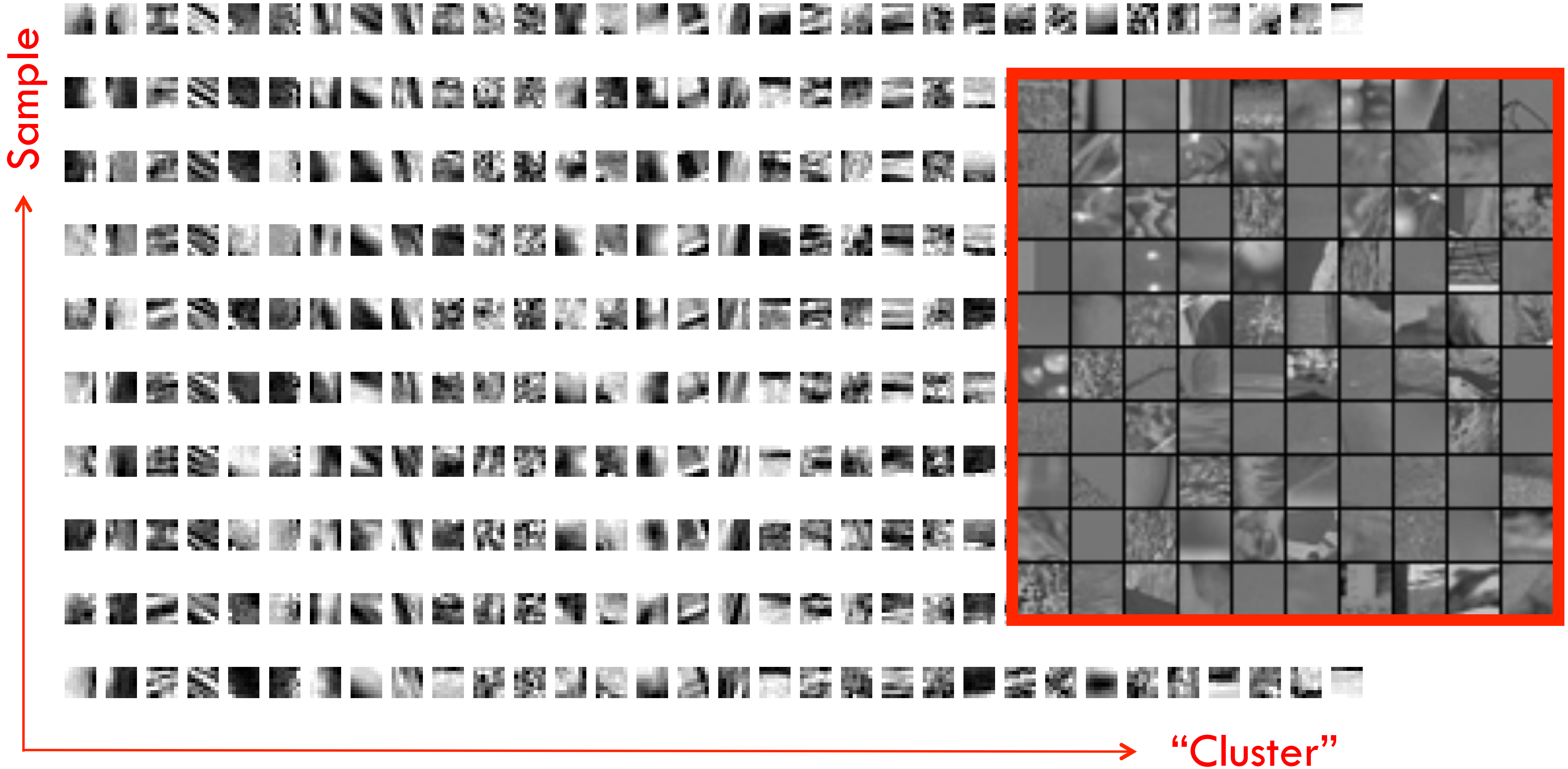
Natural Patches



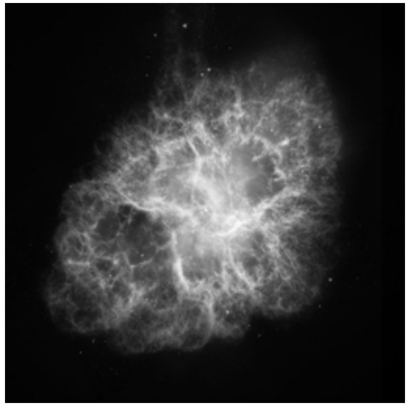
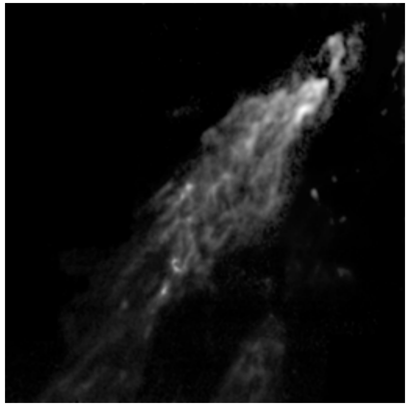
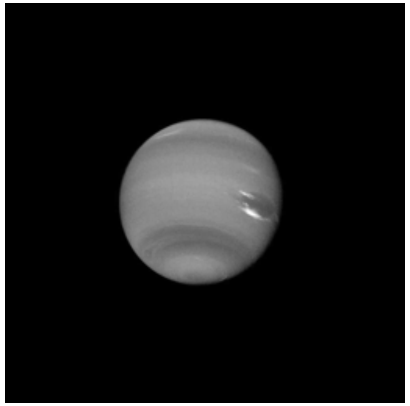
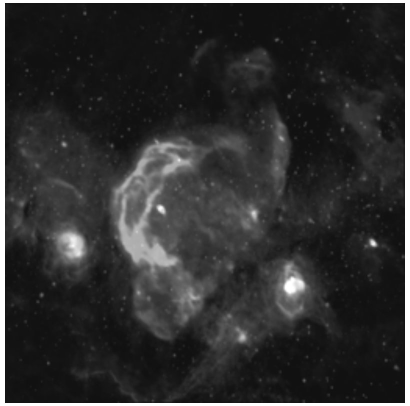
Modeling the Patches

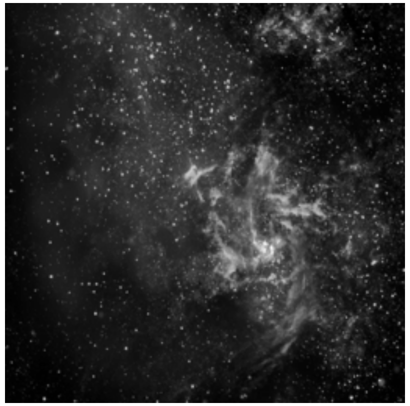
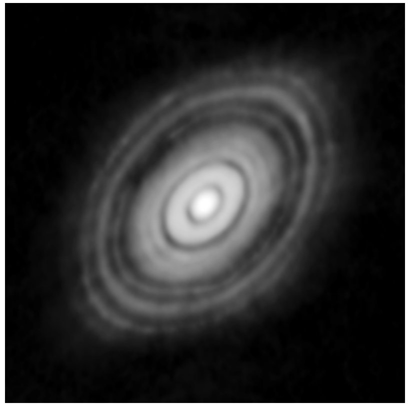
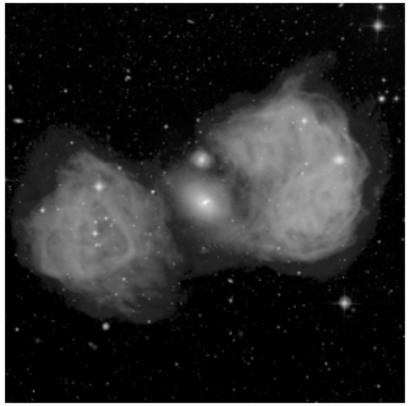
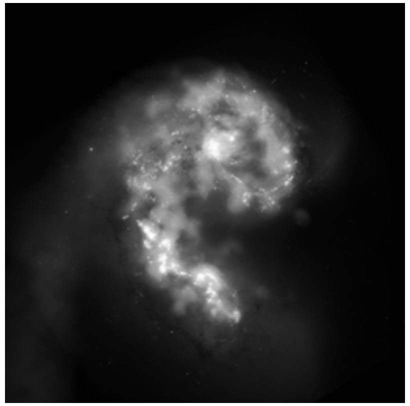


Samples from Natural Patch Model



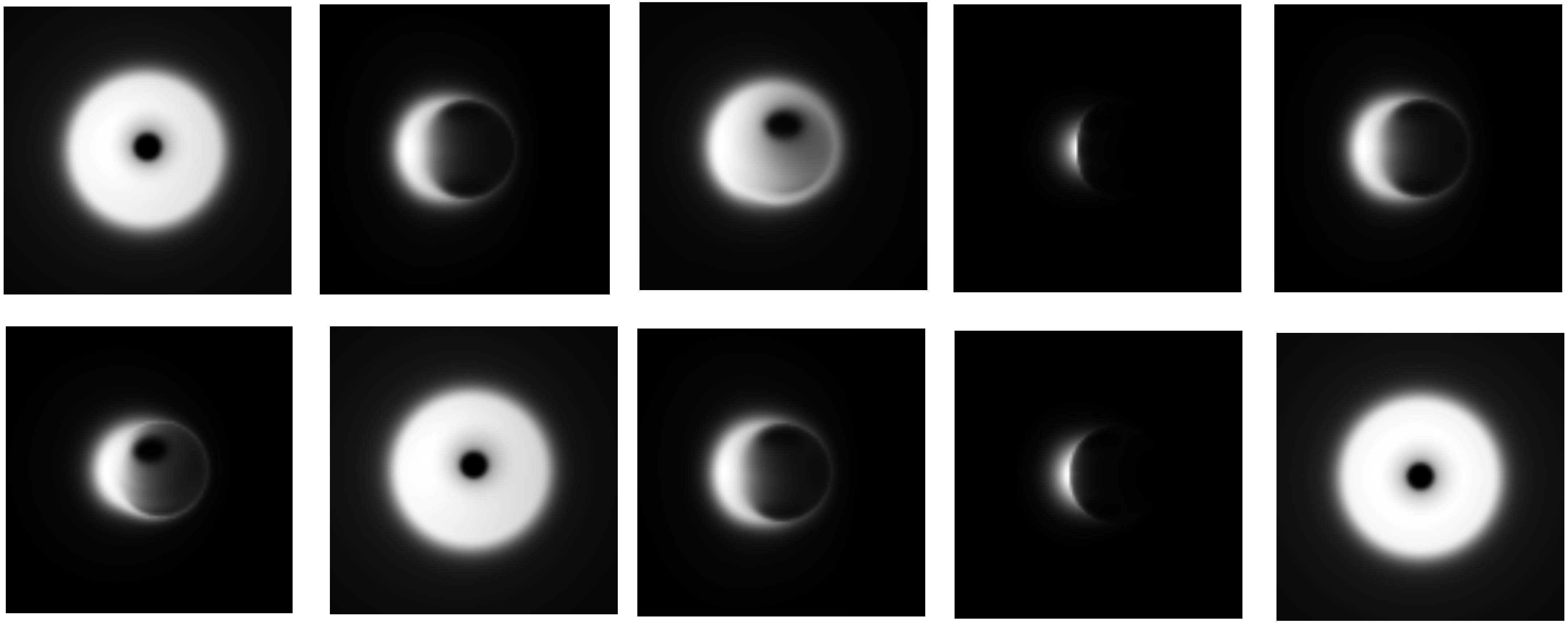
Celestial Images

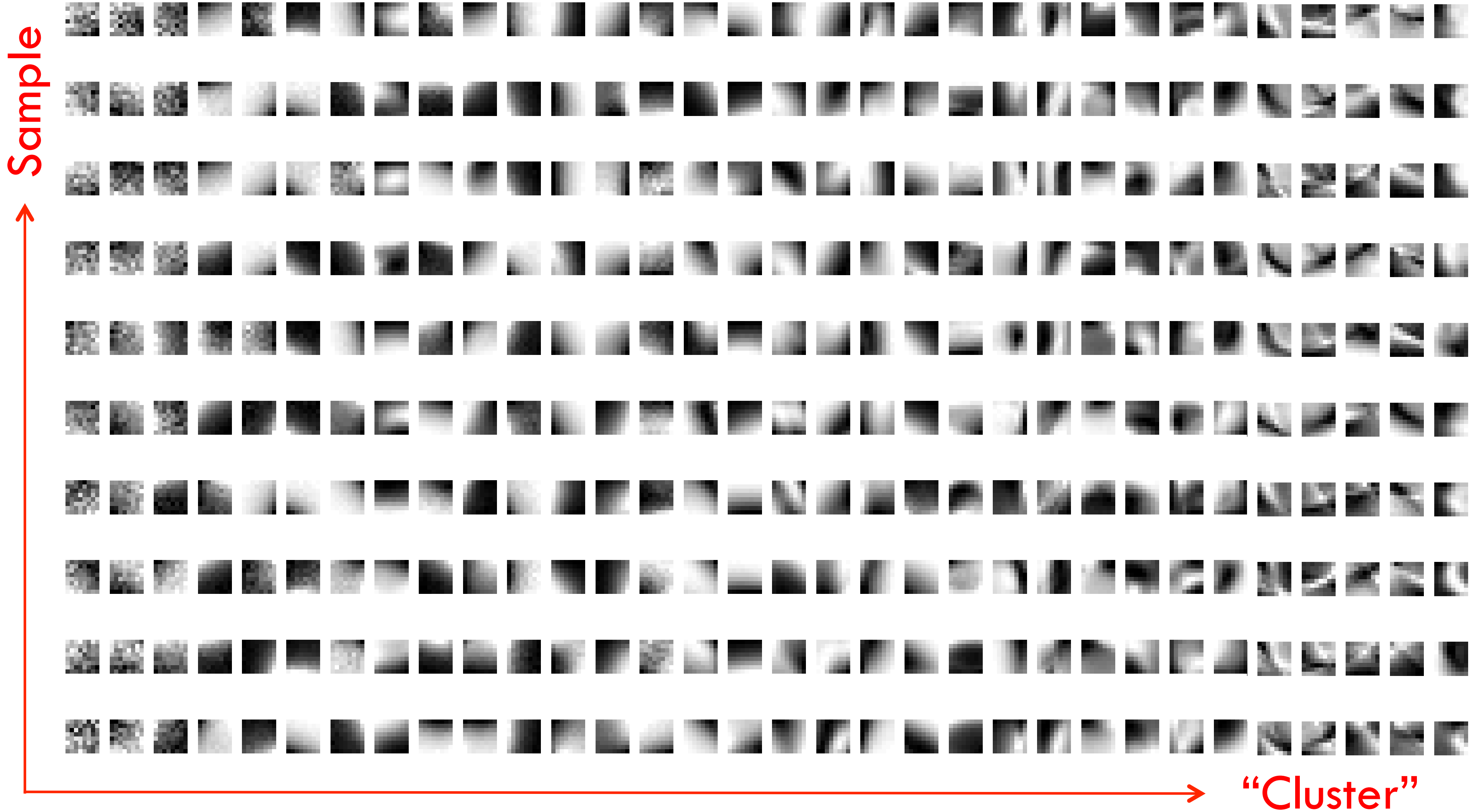

Samples from Celestial Patch Model

Black Hole Images



Images courtesy of Avery Brodrick

Samples from Black Hole Patch Model

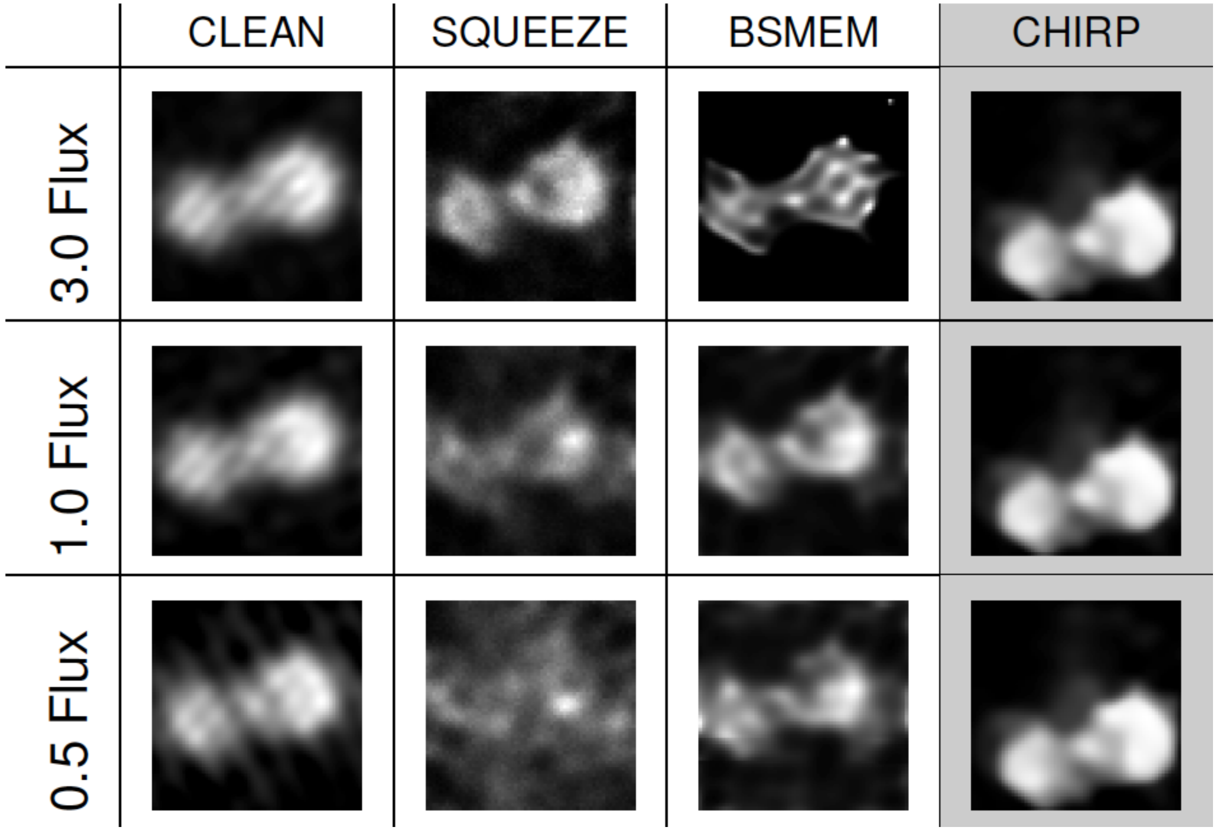


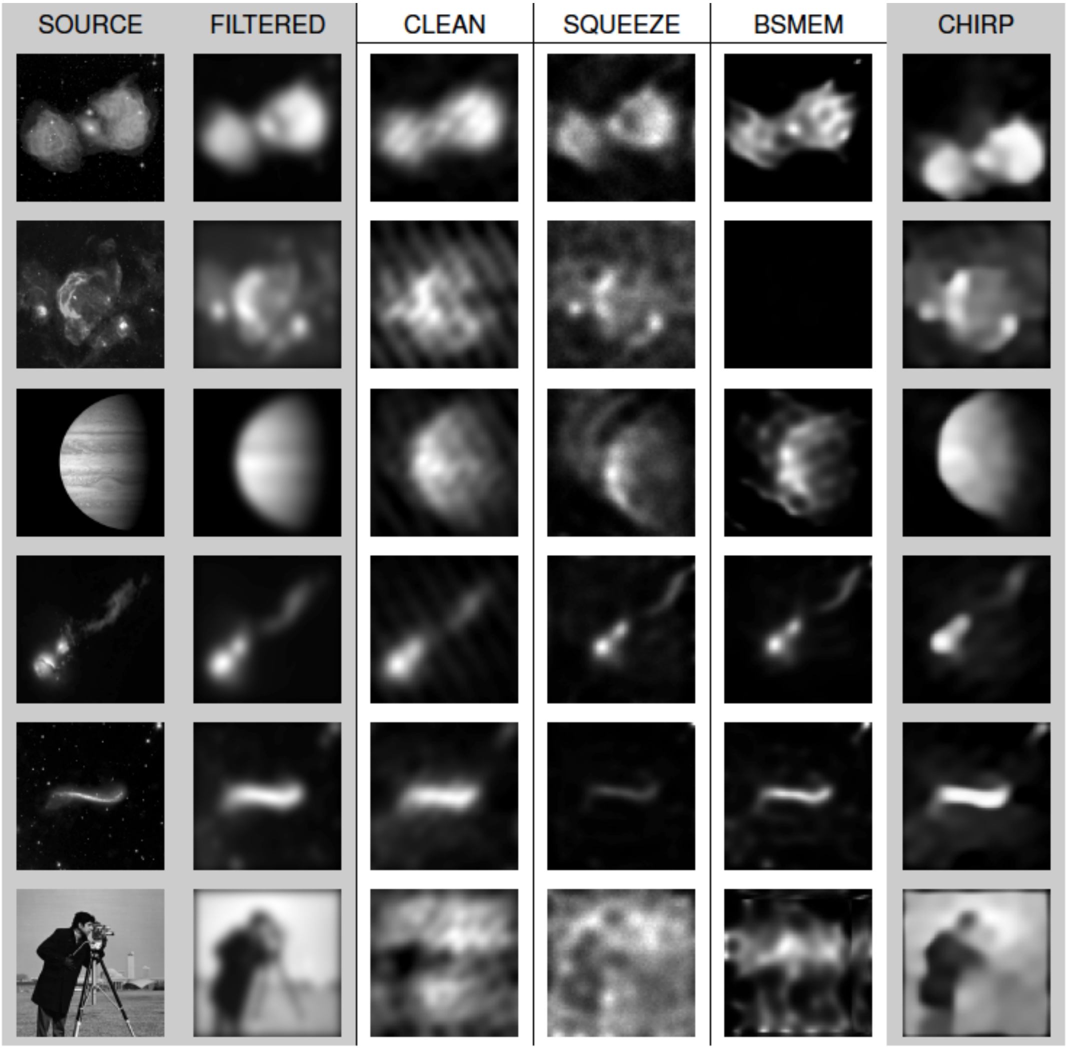
Optimization

Expected Log Likelihood - EPLL

“Half-Quadratic Splitting”

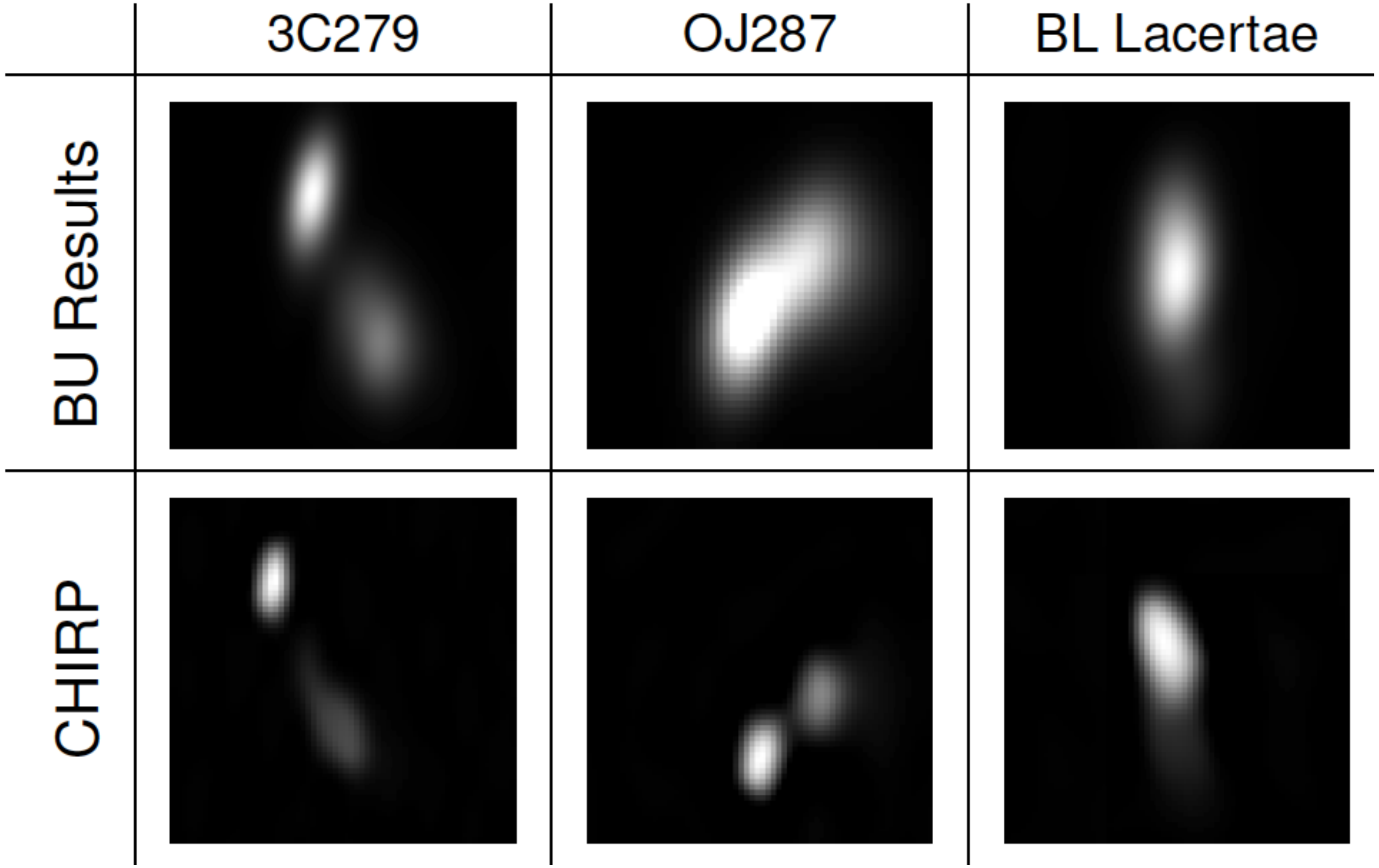
Results – Synthetic Data





Since these images were generated, we have found better parameters to use in SQUEEZE

Results – Real Data



VLBI Dataset Website

VLBI Reconstruction Dataset

A Dataset Designed to Train and Test Very Long Baseline Interferometry Image Reconstruction Algorithms

HOME | FAQ | TRAINING DATA | REAL DATA | TEST DATA | SCOREBOARD | RESULT GALLERY | GENERATE YOUR DATA

Welcome to the VLBI Reconstruction Dataset!

The goal of this website is to provide a testbed for developing new VLBI reconstruction algorithms. By supplying a large set of easy to understand training and testing data, we hope to make the problem more accessible to those less familiar with the VLBI field. Specifically, this website contains a:

- Large set of synthetic training data for many different VLBI arrays and targets
- Set of real data measurements provided in the same standard format
- Standardized data set for testing VLBI Image Reconstruction Algorithms
- Online quantitative evaluation of algorithm performance on simulated testing data
- Qualitative comparison of algorithm performance on the reconstruction of real data
- Online form to easily simulate realistic data using your own image and telescope parameters

vlbiimaging.csail.mit.edu

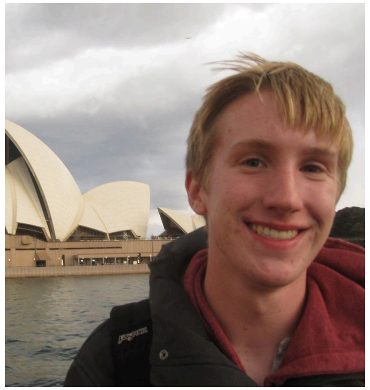
Questions?

Katie Bouman

Daniel Zoran

Bill Freeman

Michael Johnson



Andrew Chael

Vincent Fish

Sheperd Doeleman

Approximate Continuous Image: 1D

Discrete Number of Scaling Terms

$$I_\lambda(l) \approx \sum_{i=0}^{N_l - 1} X[i] h(l - (\Delta_l i + s_l))$$

Shifted Continuous Pulses

Discrete Summation

Approximate Van Cittert-Zernike Theorem: 1D

$$\Gamma(u) = \int_l e^{-i2\pi ul} I_\lambda(l)\, dl$$

$$\Gamma(u) \approx \int_l e^{-i2\pi ul} \sum_{i=0}^{N_l-1} X[i]\, h(l - (\Delta i + s_l))\, dl$$

$$e^{-i2\pi u(\Delta i + s_l)} H(u)$$

Fourier Transform of Shifted Pulse

Shift Theorem

Fourier Transform of Pulse

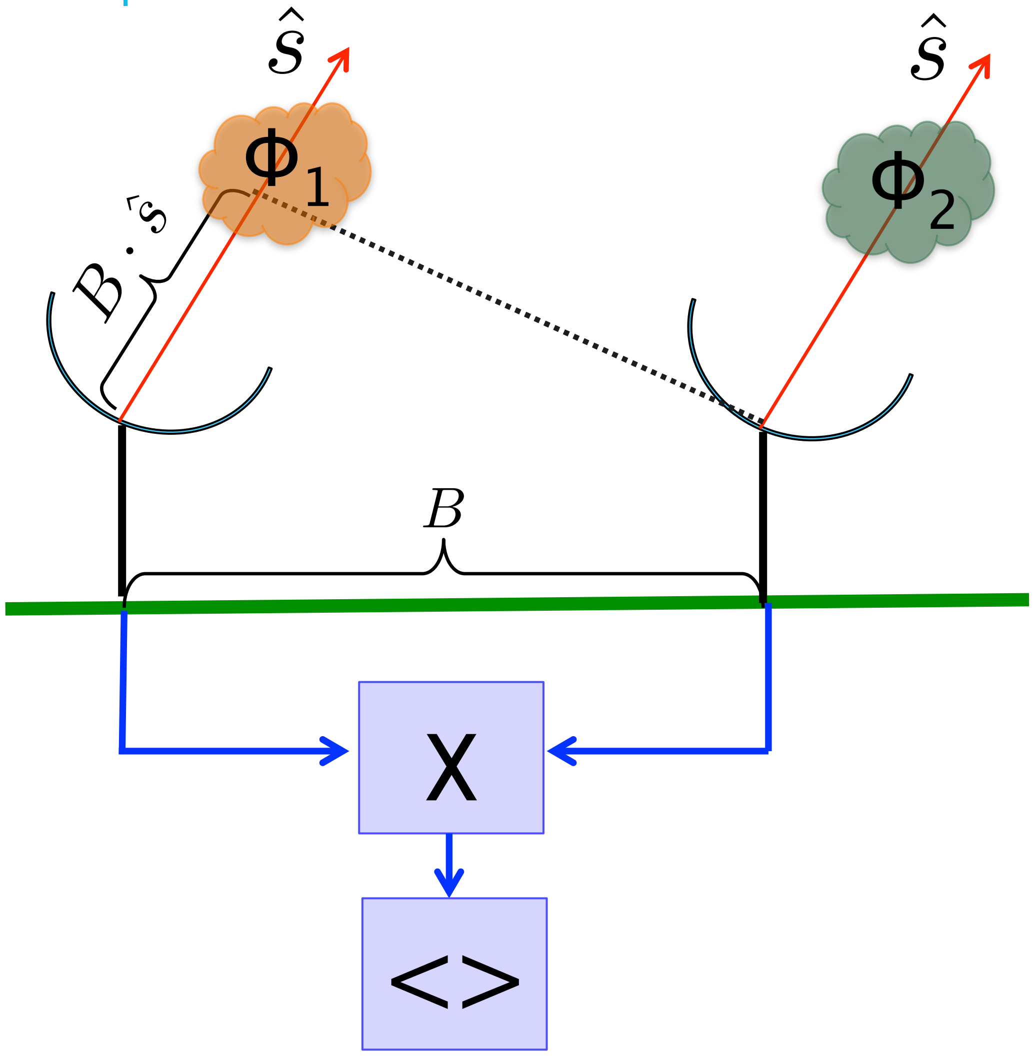
Probability of each measurement given image

$$p_{y|x}(Y|X) = \prod_{i=1}^{k} p_{y|x}(Y_i|X) = \prod_{i=1}^{k} \frac{1}{(2\pi\sigma_i^2)} \exp\left[-\frac{1}{2\sigma_i^2}||Y_i - A_i X||^2\right]$$

Probability of all measurements given image

Row vector that extracts frequency component i out of image X

Atmospheric Noise and Closure Phase



$$\omega\tau_{1,2} + \phi_1 - \phi_2$$: Telescopes 1 x 2

$$\omega\tau_{2,3} + \phi_2 - \phi_3$$: Telescopes 2 x 3

$$+ \; \omega\tau_{3,1} + \phi_3 - \phi_1$$: Telescopes 3 x 1

$$\omega\tau_{1,2} + \omega\tau_{2,3} + \omega\tau_{3,1}$$

Overview

Image Reconstruction Algorithm

Likelihood
"Data Term"

- Image Representation
- **Bispectrum Energy**

Prior
"Previous Expectations Term"

- Training a Patch Prior
- Reconstructing with a Patch Prior

Overview

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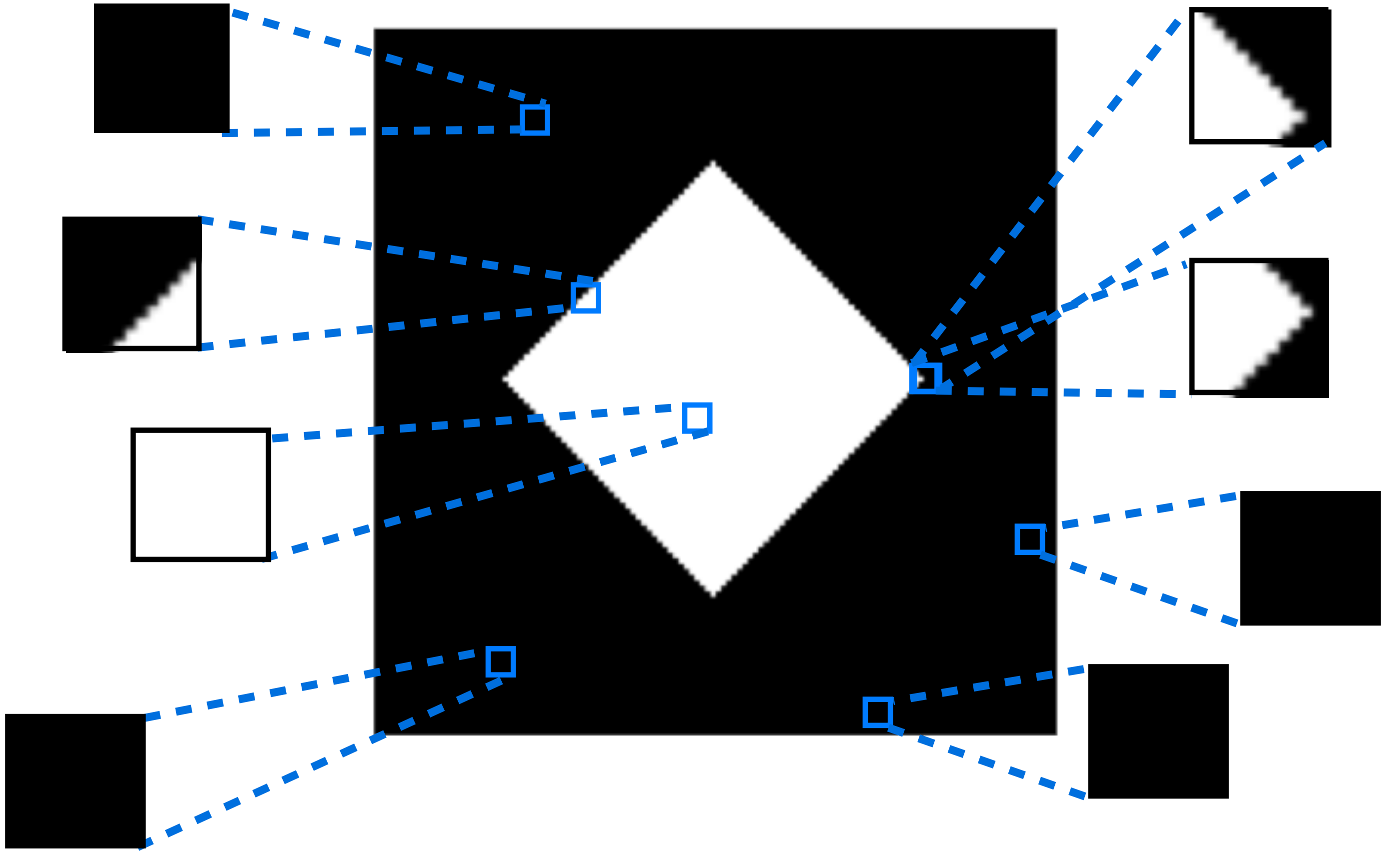
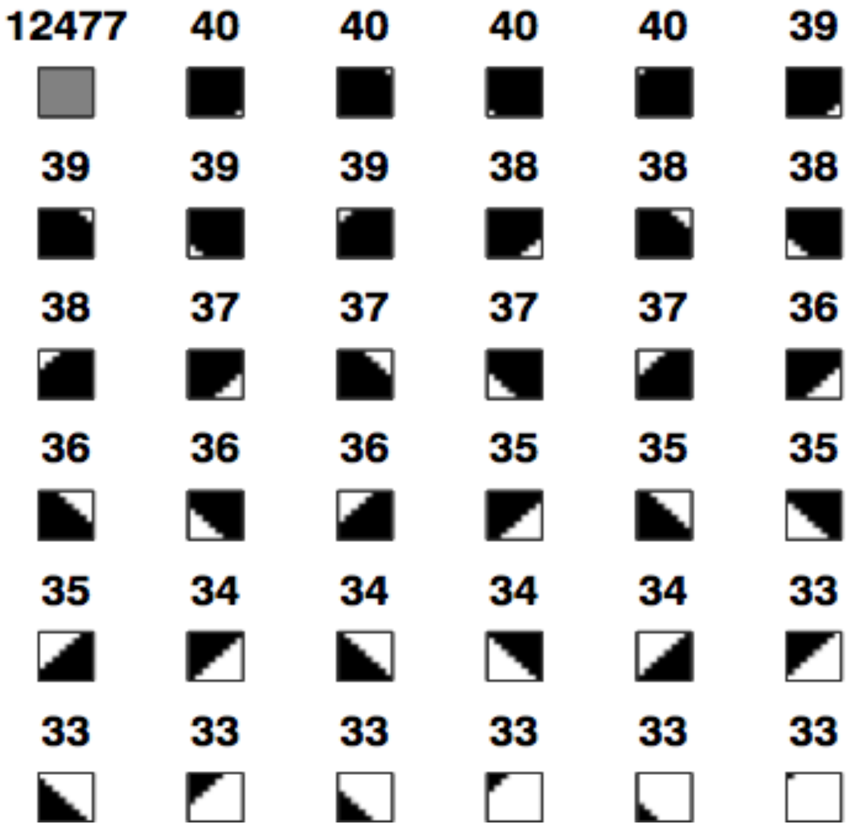
Simple Example



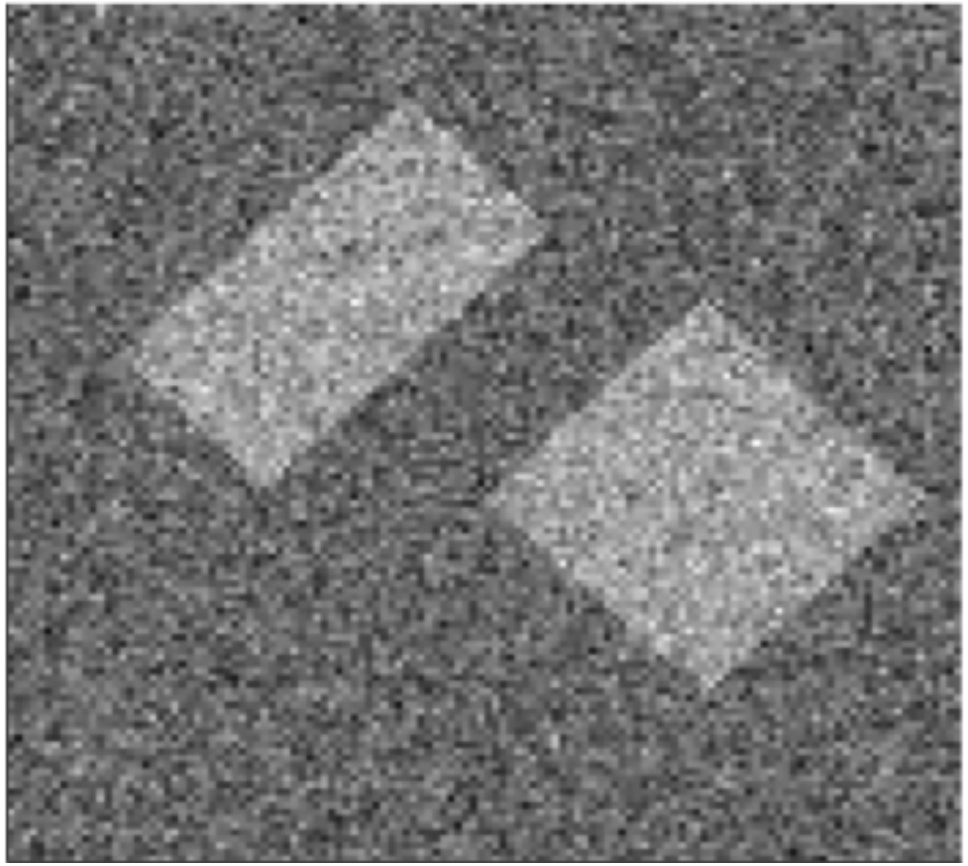
Training Image

Simple Example



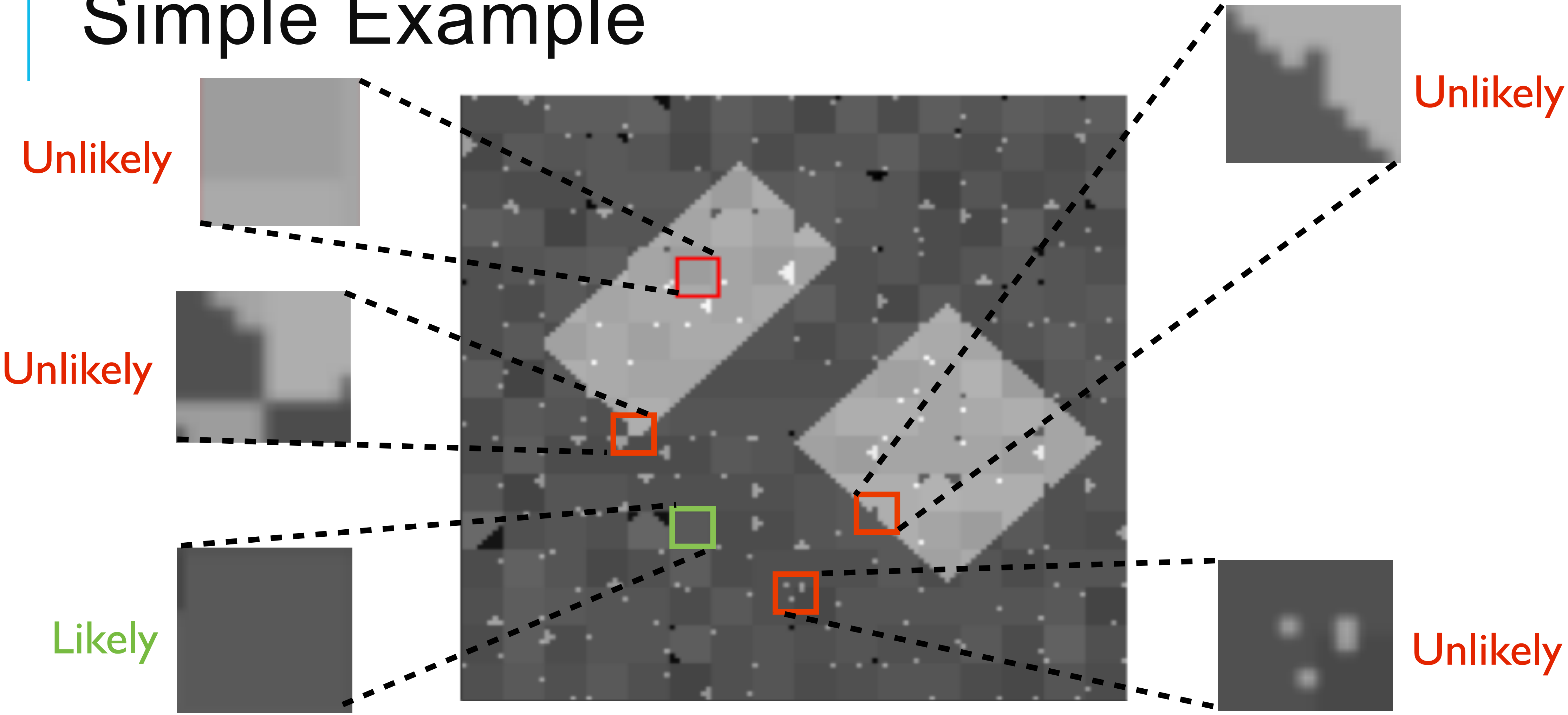
A Simple Prior Learned from Training Data

Simple Example



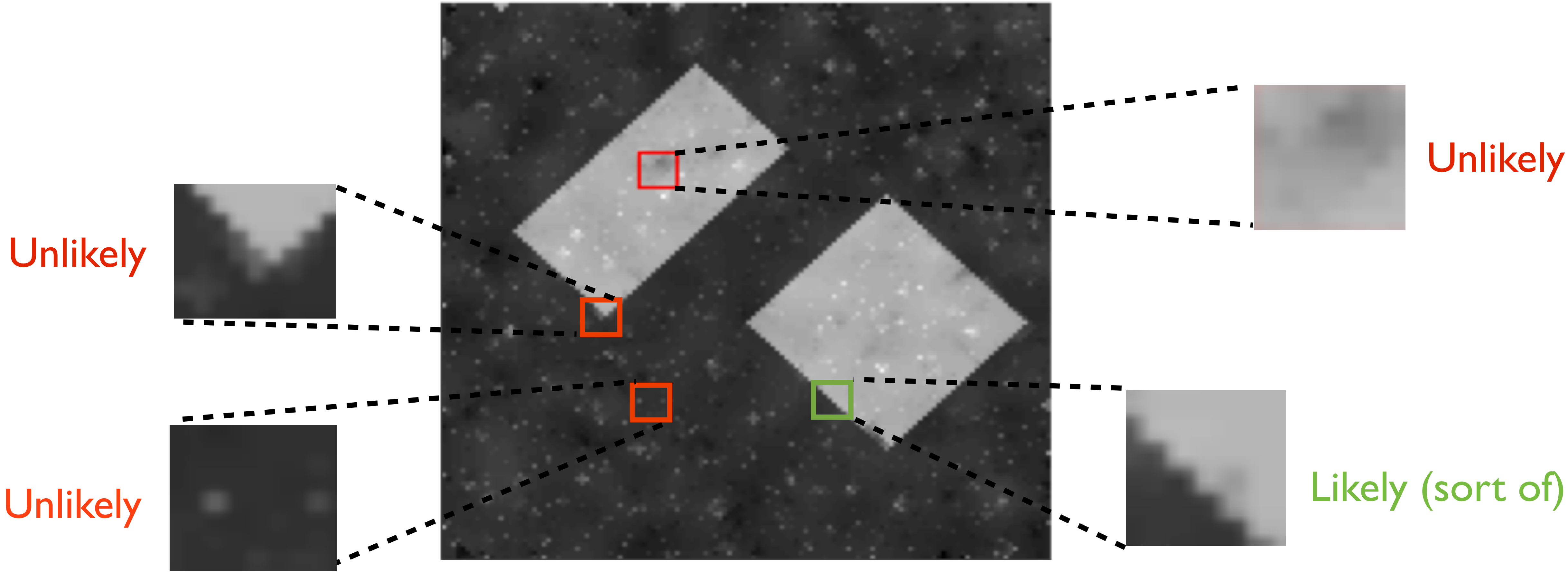
Noisy image we wish to restore using our patch prior

Simple Example



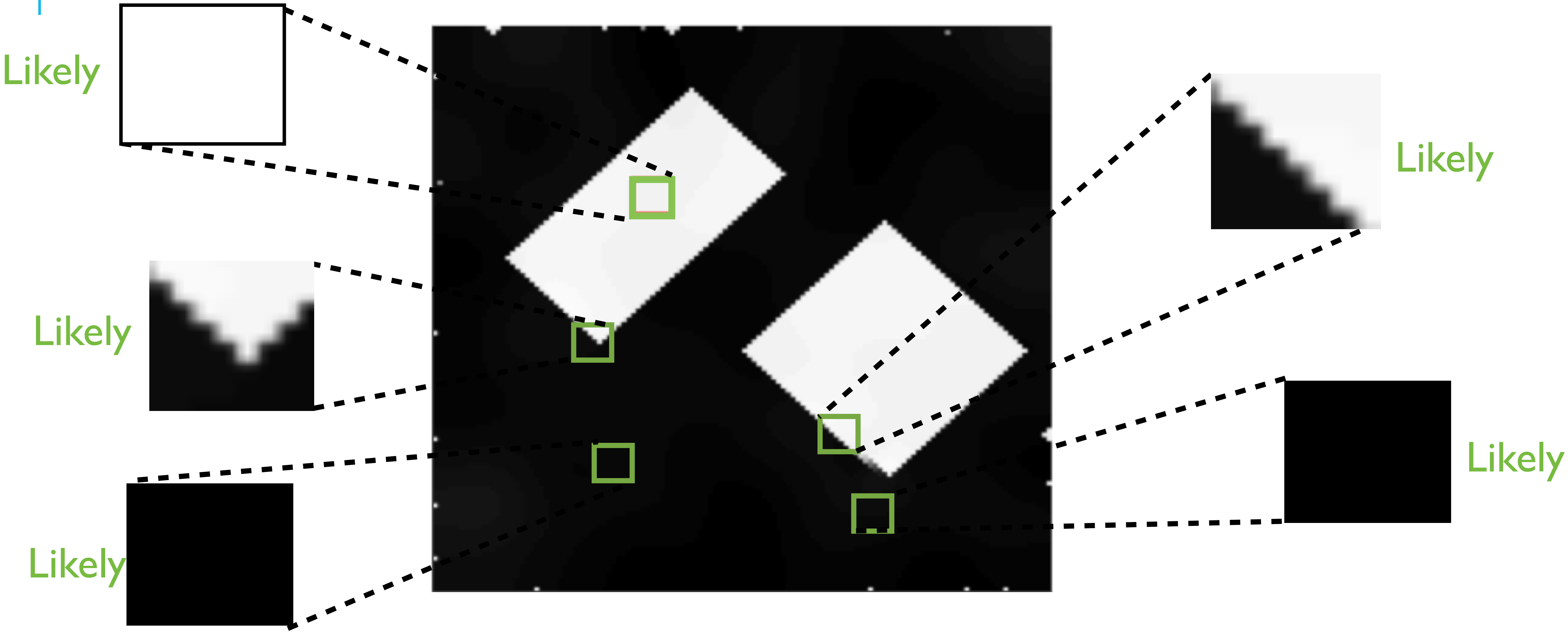
Non-Overlapping Patches

Simple Example



Overlapping Patches - Patch Averaging

Simple Example



We want every patch in the output to be likely

Expected Patch Log Likelihood - EPLL

We propose the EPLL cost function:

EPLL is NOT P(x)

Learning with patches, not images! (as opposed

$$f_p(\mathbf{x}|\mathbf{y}) = \frac{\lambda}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|^2 - \sum_i \log p(\mathbf{P}_i\mathbf{x})$$

Optimization

We use "half-quadratic splitting"

Introduce a set of auxiliary variables **Z**

Solve the following optimization problem:

$$c_{p,\beta}(\mathbf{x}, \mathbf{Z}|\mathbf{y}) = \frac{\lambda}{2}||\mathbf{A}\mathbf{x} - \mathbf{y}||^2 +$$