Fornax A - Credit: Credit: NRAO/AUI/NSF

Advanced imaging

European Radio Interferometry School 2022

Dr Jack F. Radcliffe (U. Pretoria / U. Manchester)
Dr Cristiana Spingola (INAF)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

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Outline

1. Recap of imaging
2. Other deconvolution algorithms (e.g., multi-frequency, multi-scale)
3. Wide-field imaging and direction dependent effects

Recap of imaging

- After 'perfect calibration' the visibilities are represented by the following equation,

$$\mathsf{V}(u,v,w) = \iint_{lm} \frac{\mathsf{B}(l,m)}{n} \exp\left\{-2\pi i \left[ul + vm + w\,(n-1)\right]\right\} \mathrm{d}l\mathrm{d}m$$

where $n = \sqrt{1 - l^2 - m^2}$

- Here,
  - $(u,v,w) =$ interferometer geometric vector
  - $(l,m) =$ directional cosines / sky coordinates
  - $\mathsf{B}(l,m) =$ sky brightness / intensity distribution

With imaging, **we want to calculate** $\mathsf{B}(l,m)$ **from** $\mathsf{V}(u,v)$

Recap of imaging

$$\mathsf{V}(u,v,w) = \iint_{lm} \frac{\mathsf{B}(l,m)}{n} \exp\left\{-2\pi i\left[ul + vm + w\,(n-1)\right]\right\}\,\mathrm{d}l\mathrm{d}m$$

where $n = \sqrt{1 - l^2 - m^2}$

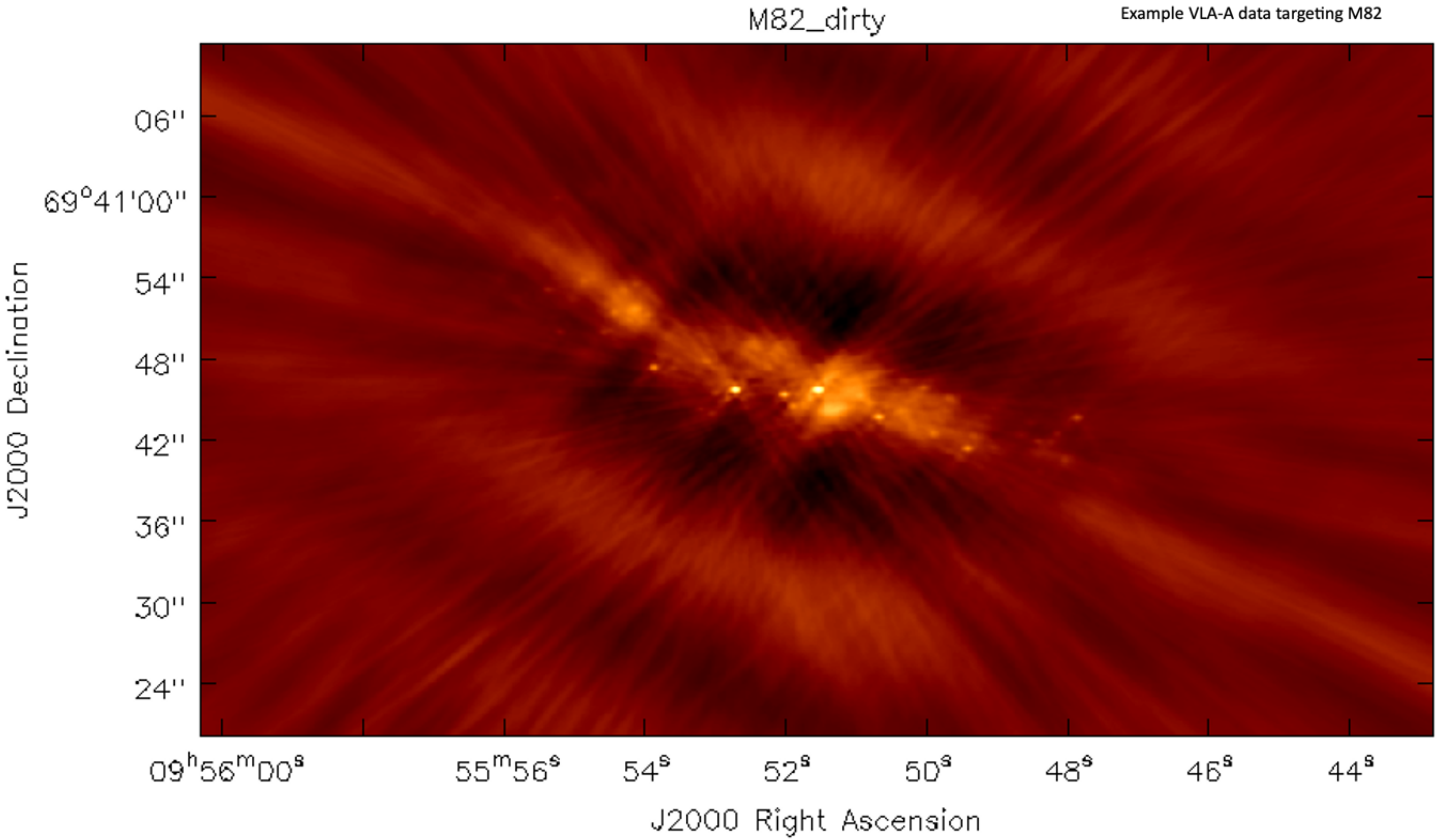
- In the imaging lecture / workshop, we made the assumption that we are observing a small field-of-view ($l, m \rightarrow 0$) or we have short baselines ($w \rightarrow 0$). This means that we can omit the w term and the equation becomes,

$$\mathsf{V}(u,v) \approx \iint_{l,m} \mathsf{B}(l,m)\exp\left\{-2\pi i\left[ul + vm\right]\right\}\,\mathrm{d}l\mathrm{d}m$$

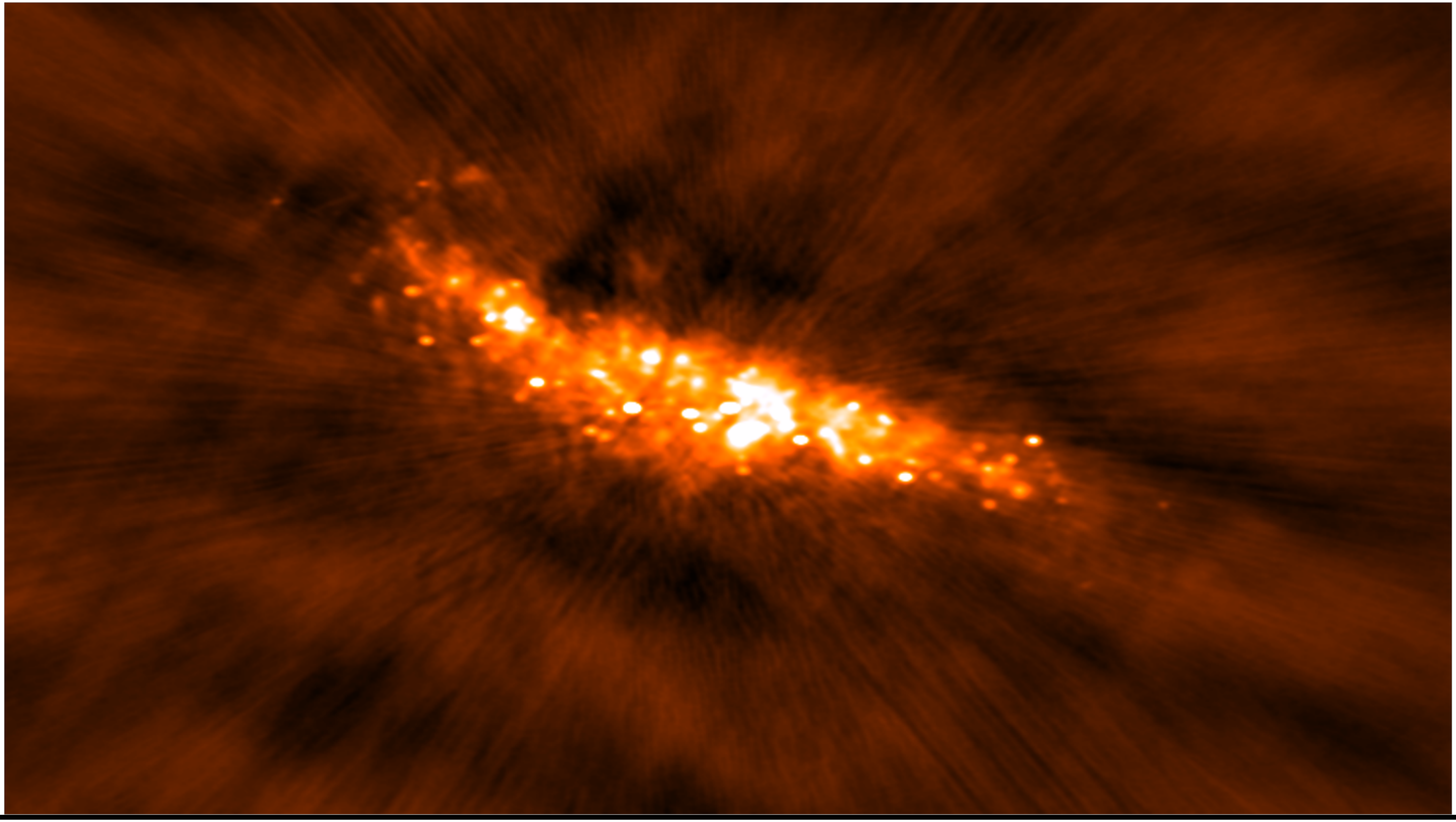
- **Relationship between $\mathsf{V}(u,v)$ and $\mathsf{B}(l,m)$ is?**

Dirty image

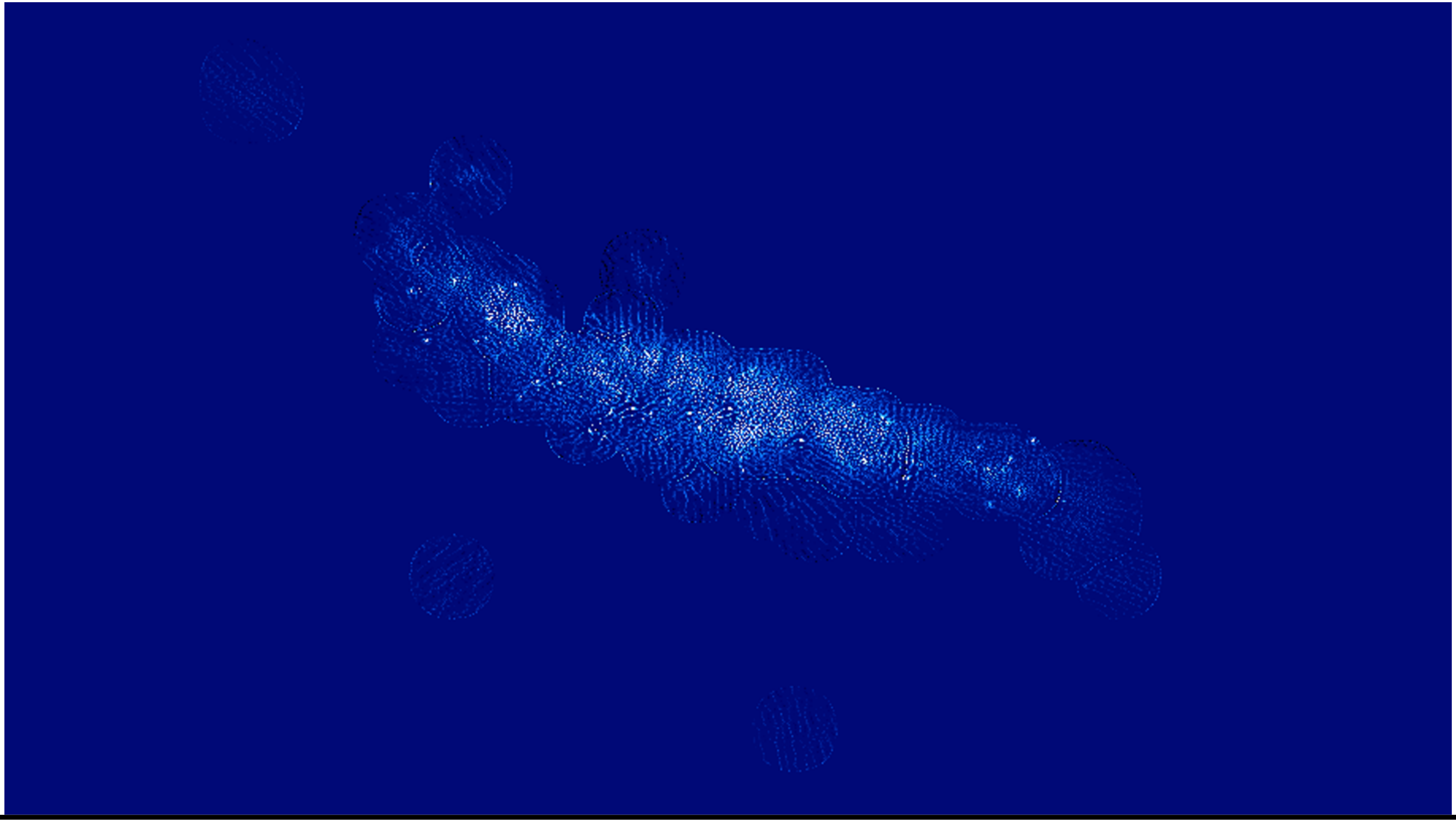
Example VLA-A data targeting M82



Högbom deconvolution



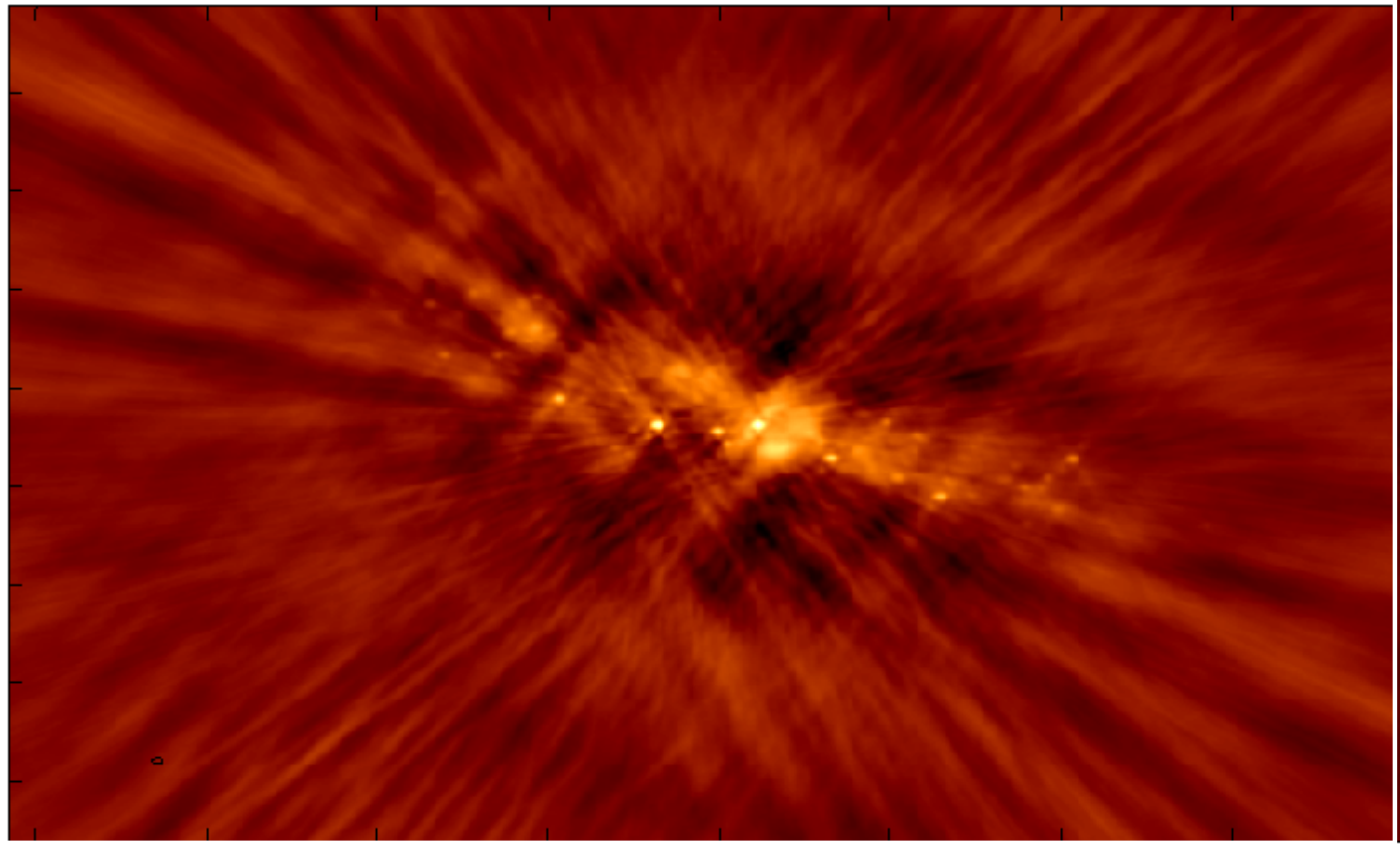
Högbom deconvolution

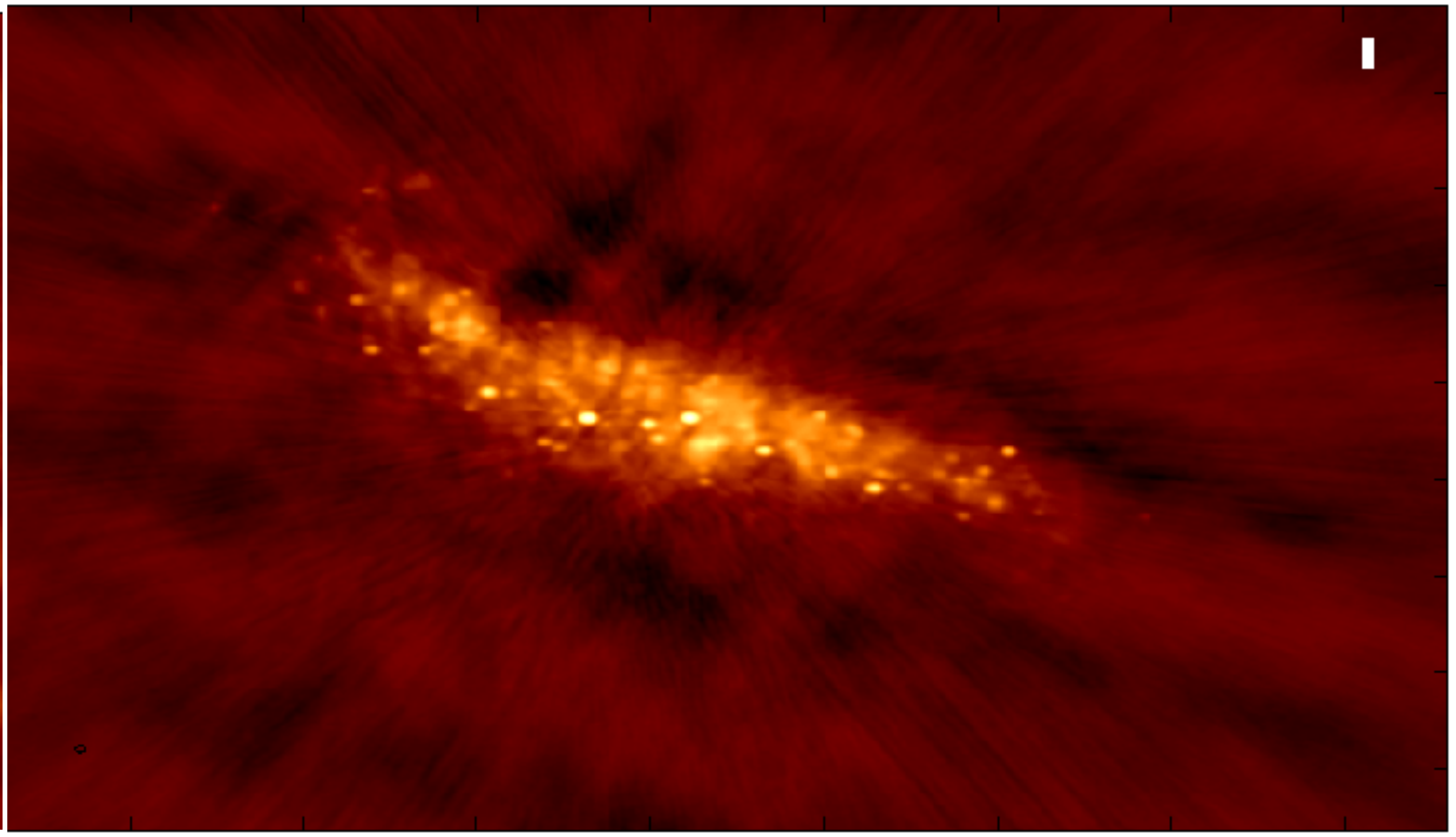


Many forms of CLEAN

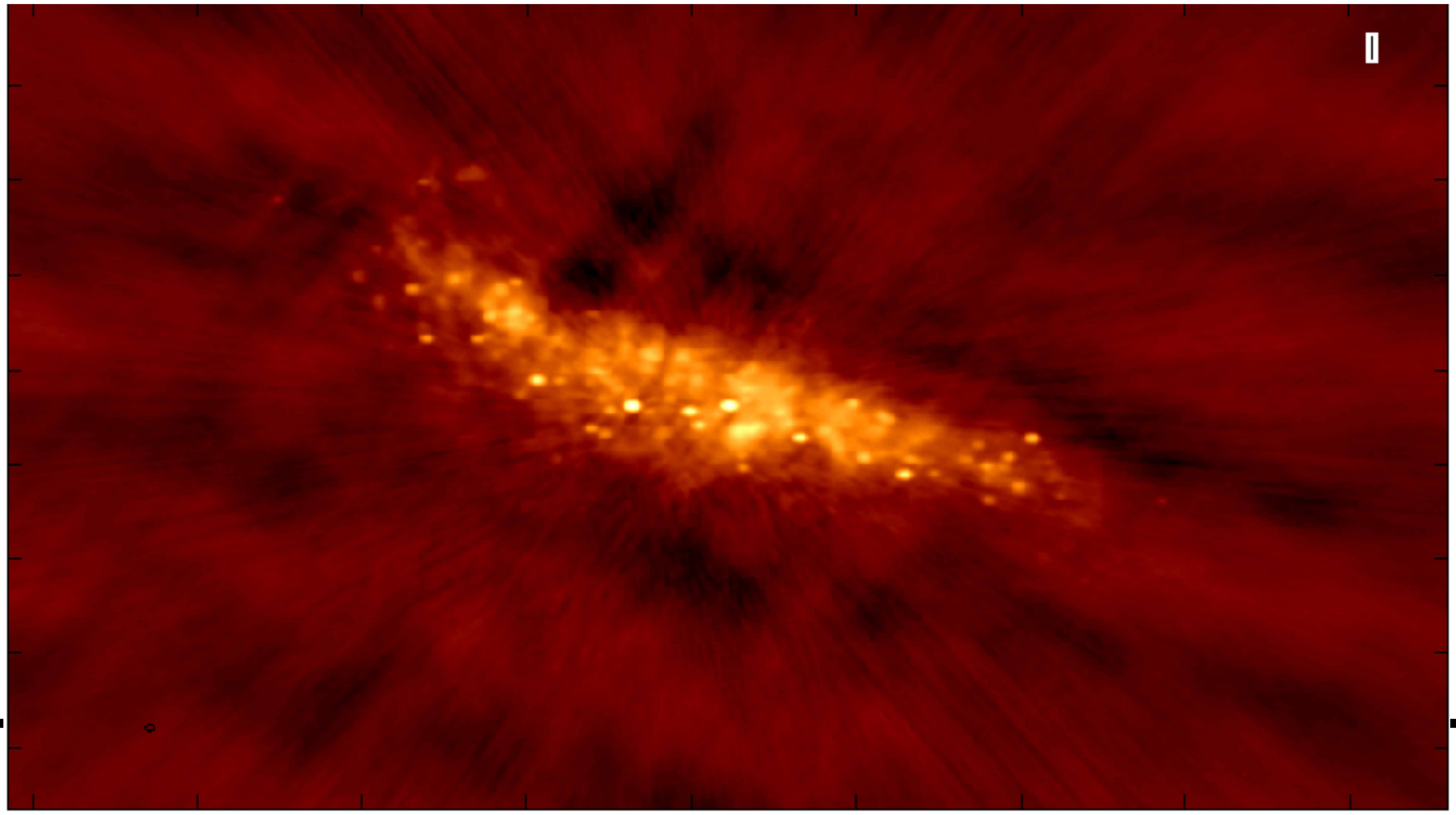
Clark

Maximum Entropy Method





Clark-Stokes



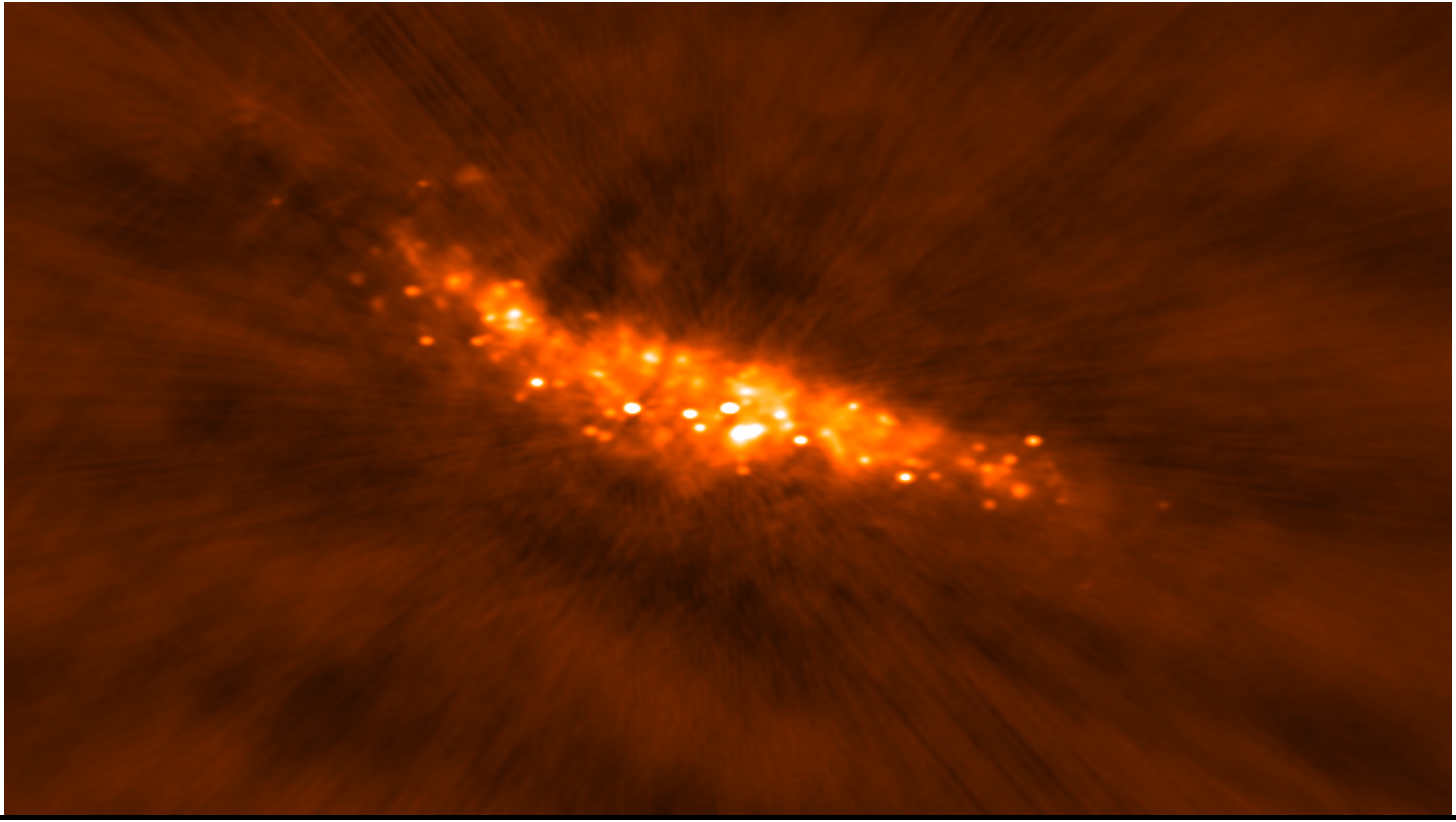
Deconvolving diffuse structure - multiscale

- Improved algorithm by Cornwell (2008) : “multi-scale clean”
- Fits **small smooth Gaussian kernels** (and delta functions) during a Högbom CLEAN iteration
- Implemented in CASA tclean. Advised to use pixel scales corresponding to orders of the dirty beam size and avoid making scale too large compared to the image width/lowest spatial frequency.

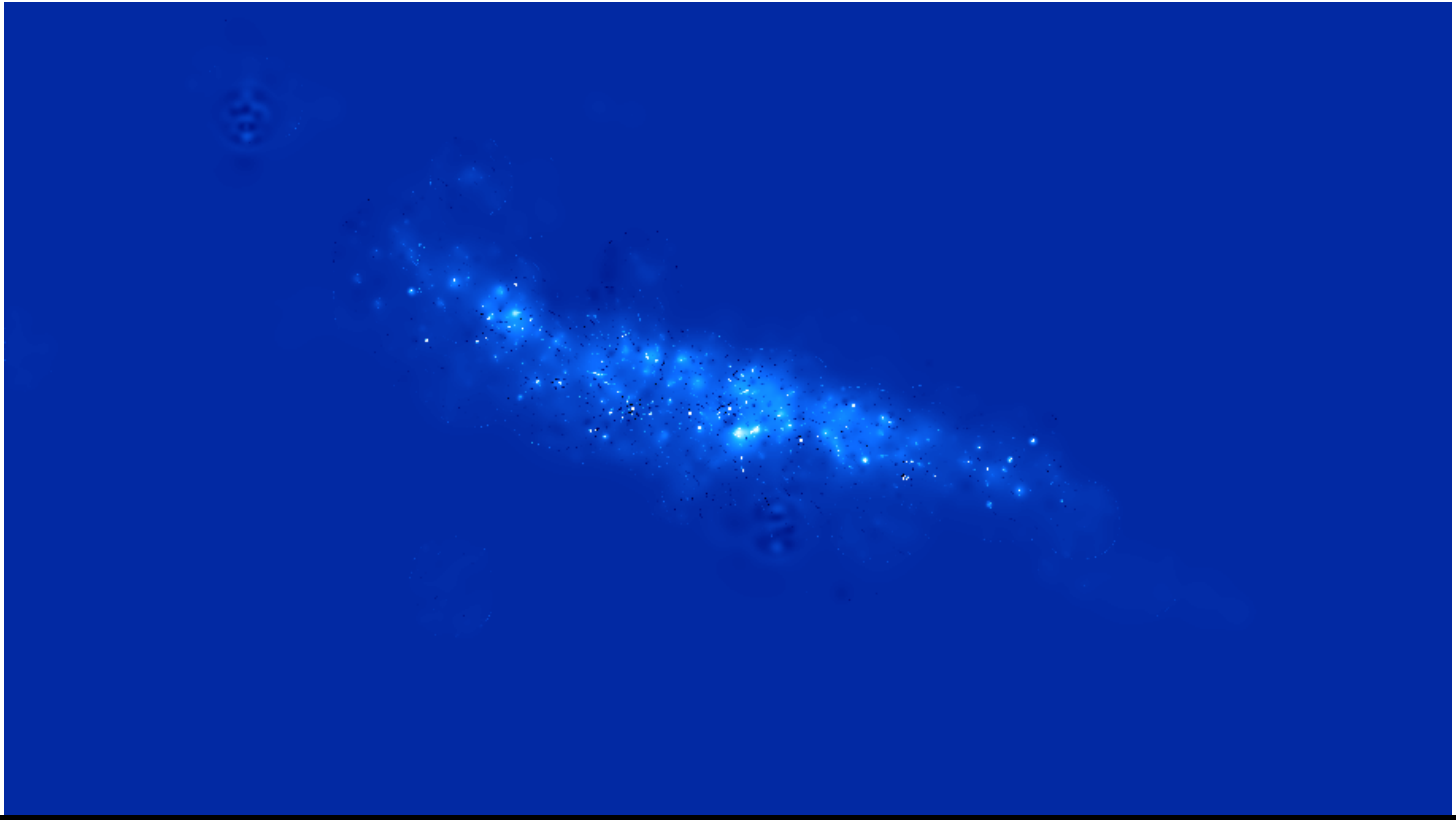
```
deconvolver        = 'multiscale'     #  Minor cycle algorithm (hogbom,clark,m
                                      #   ultiscale,mtmfs,mem,clarkstokes)
     scales        = [0, 1, 5, 15]    #  List of scale sizes (in pixels) for
                                      #   multi-scale algorithms
     smallscalebias =        0.6      #  A bias towards smaller scale sizes
     restoringbeam  =         []      #  Restoring beam shape to use. Default
                                      #   is the PSF main lobe
```

CASA tclean

Multi-scale image



Multi-scale model



Dealing with wide-bandwidths - mtmfs

- Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid.
- Done automatically to improve *uv* coverage.

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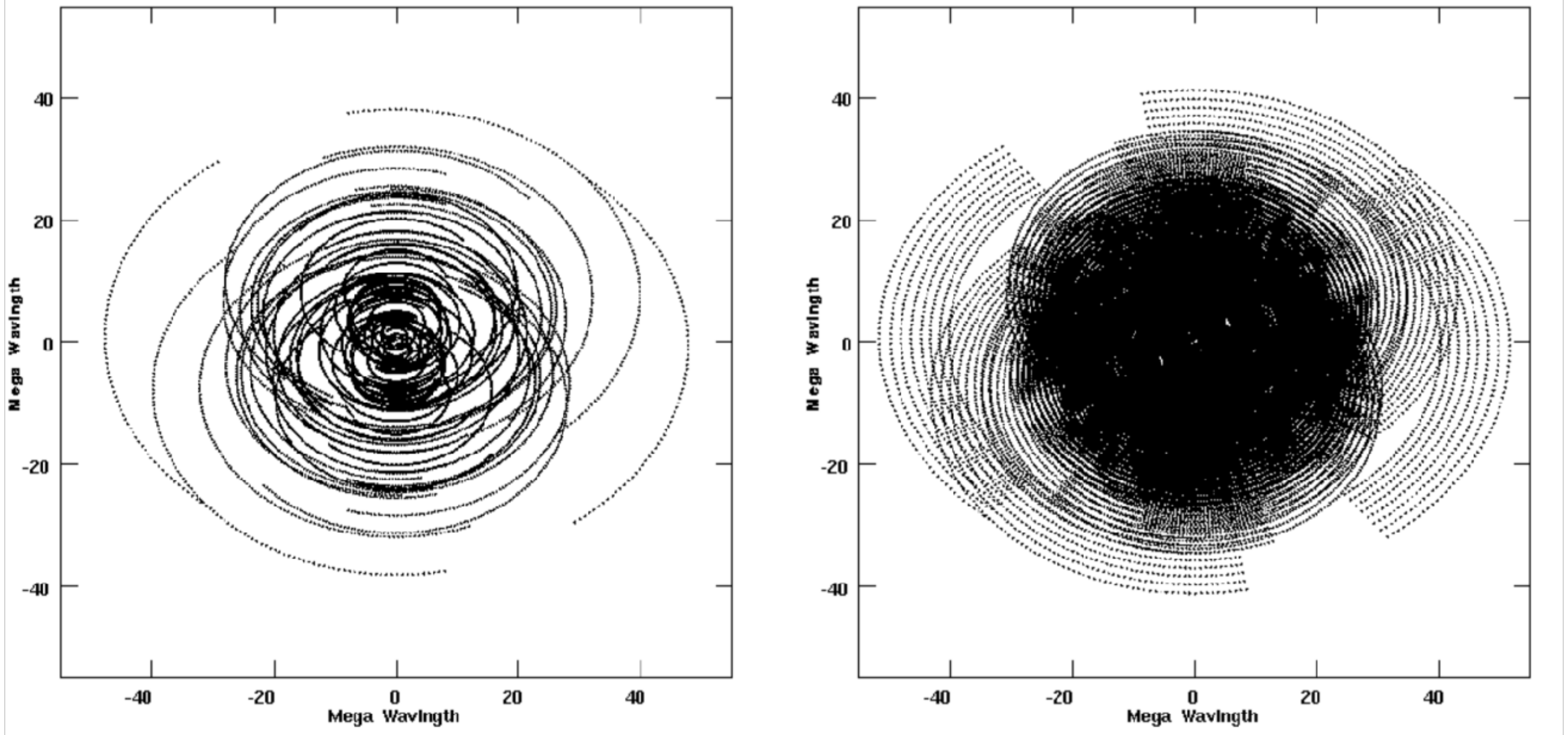
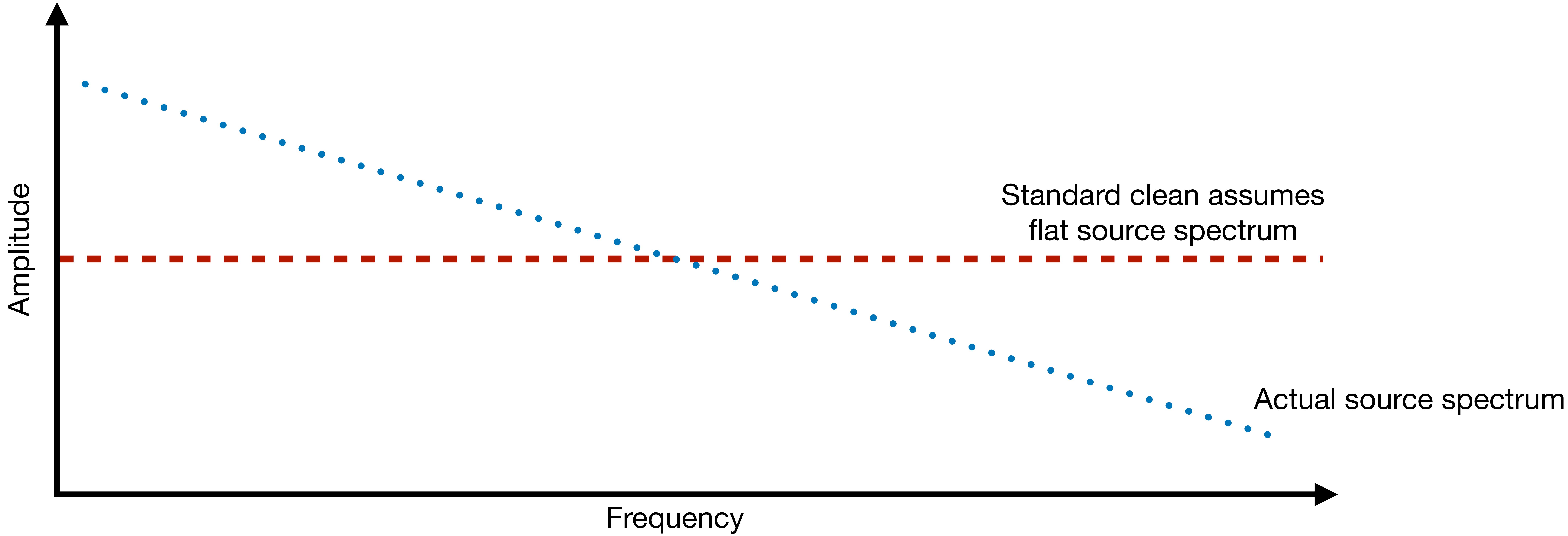


Figure 16.1: *Left (a):* VLBA (u, v) coverage for a full track at $\delta = 50°$. *Right (b):* Using MFS observations with 8 frequencies spread over 25%.

Multi-frequency deconvolution

- But there is a problem if the source flux density changes with frequency…
- Need what is called multi-term multi-frequency synthesis (MTMFS) imaging.
- Takes spectral variation of sky brightness distribution into account during deconvolution using linear Taylor series approximation.



Multi-frequency deconvolution

- $\mathsf{B}_\nu(l, m)$ represents the sky emission in terms of a Taylor series about a reference frequency:

$$\mathsf{B}_\nu = \sum_{t=0}^{N_t-1} b_\nu^t \mathsf{B}_t^{\mathrm{sky}} \text{ where } b_\nu^t = \left(\frac{\nu - \nu_0}{\nu_0}\right)^t$$

- A good practical choice is a power law model of the sky:

$$\mathsf{B}_\nu^{\mathrm{sky}} = \mathsf{B}_{\nu_0}^{\mathrm{sky}} \left(\frac{\nu}{\nu_0}\right)^{\mathsf{B}_\alpha^{\mathrm{sky}} + \mathsf{B}_\beta^{\mathrm{sky}} \log\left(\frac{\nu}{\nu_0}\right)}$$

- Additional terms can be added depending on the sky model.

Sparse reconstruction - compressed sensing methods

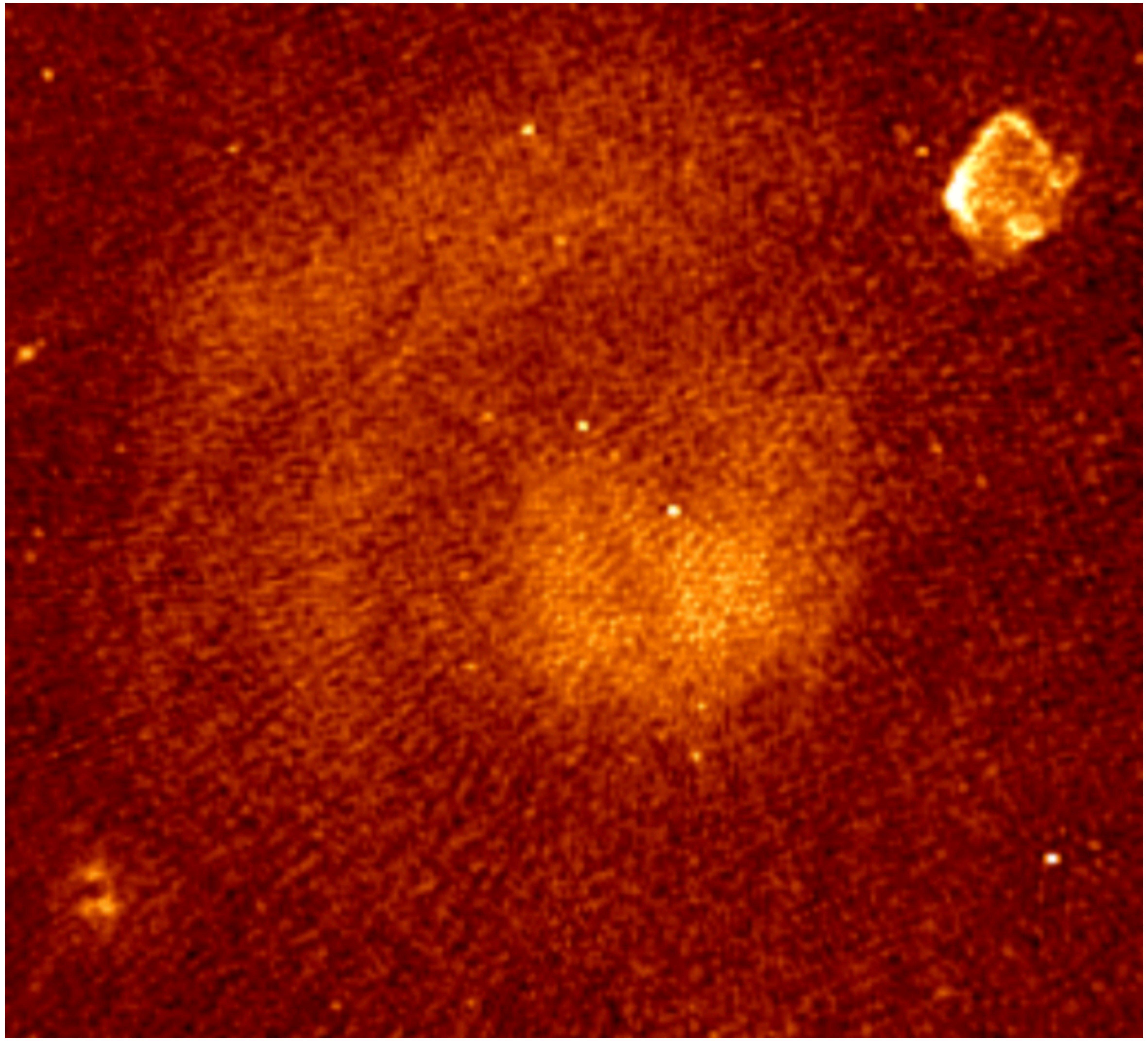


Image credit - A. Offringa

Compressed sensing

- Model with CS

Image credit - A. Offringa

Compressed sensing

- Model with multi-scale

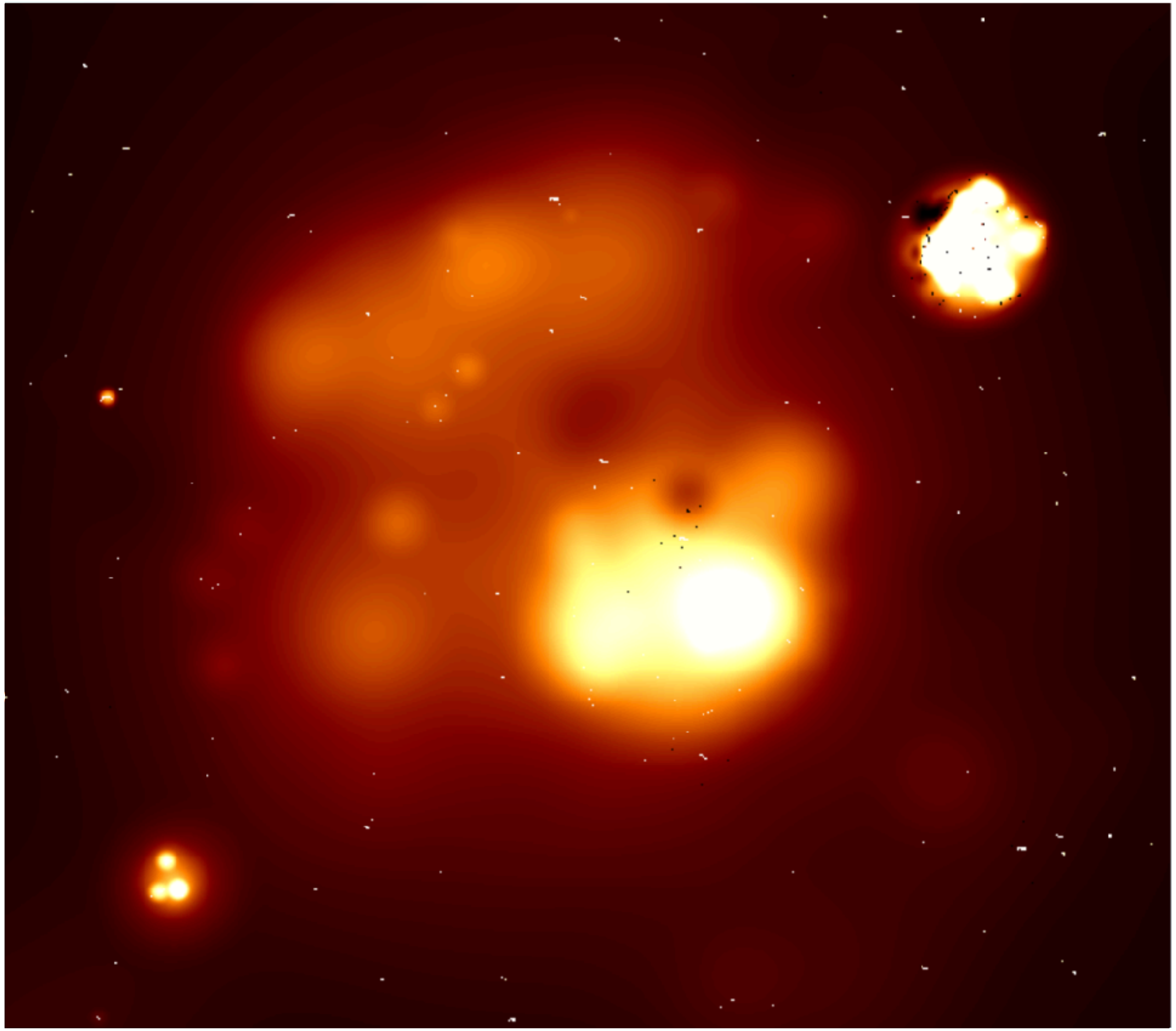


Image credit - A. Offringa

Direction-dependent effects

- To the RIME, we had the corrupting effects being parameterised as a Jones chain:

$$\mathsf{V}_{pq} = \iint_{lm} \boldsymbol{J}_p \frac{\mathsf{B}(l,m)}{n} \exp\left\{-2\pi i \left[u_{pq}l + v_{pq}m + w_{pq}(n-1)\right]\right\} \boldsymbol{J}_q^H \,\mathrm{d}l\mathrm{d}m$$

$$\mathsf{V}_{pq} = \iint_{lm} \boldsymbol{J}_p K_p \mathsf{B}(l,m) K_q^H \boldsymbol{J}_q^H \,\mathrm{d}l\mathrm{d}m$$

- Can be split into direction-independent ($\boldsymbol{G}$) and direction-dependent effects, DDEs ($\boldsymbol{E} = \boldsymbol{E}(l,m)$)

$$\mathsf{V}_{pq} = \boldsymbol{G}_p \left(\iint_{lm} \boldsymbol{E}_p K_p \mathsf{B}(l,m) K_q^H \boldsymbol{E}_q^H \,\mathrm{d}l\mathrm{d}m \right) \boldsymbol{G}_q^H$$

- These $\boldsymbol{E}$ terms causes your interferometer to effectively ‘see’ a different sky on each baseline and can cause errors. Most are calibrated away through observational design / strategies (e.g., phase referencing).
- Some are not, and can change over your field-of-view…

Wide-field imaging

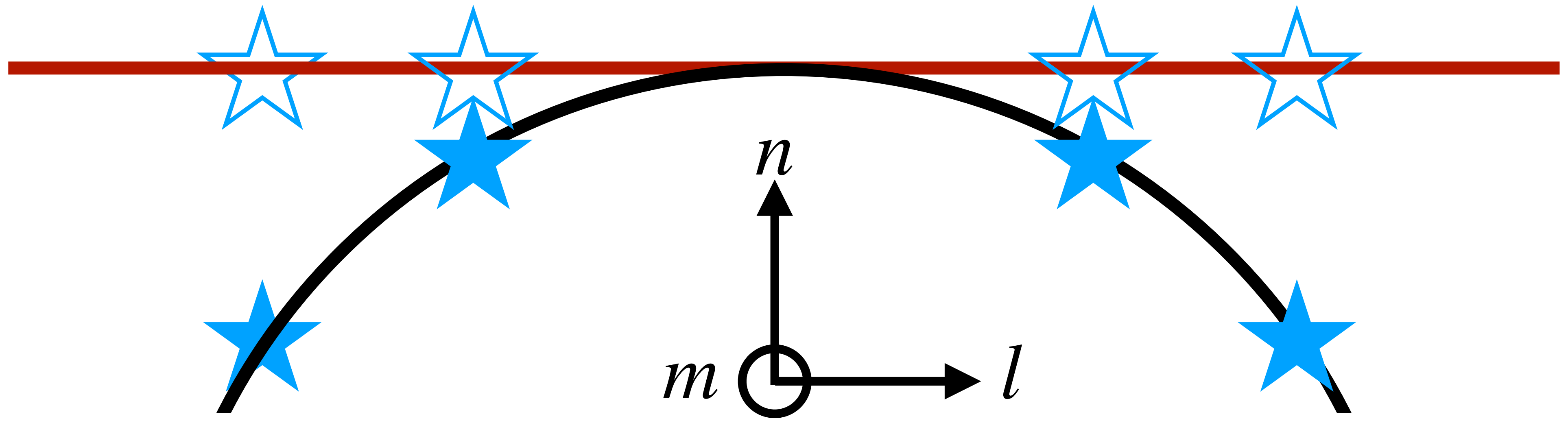
- First direction dependent effect is the non-coplanar term (or the w-term),

$$\mathsf{V}(u,v,w) = \iint_{lm} \frac{\mathsf{B}(l,m)}{n} \exp\left\{-2\pi i \left[ul + vm + w\,(n-1)\right]\right\} \mathrm{d}l\mathrm{d}m$$

$$\rightarrow \mathsf{V}(u,v,w) = \iint_{lm} W\,\frac{\mathsf{B}(l,m)}{n} \exp\left\{-2\pi i \left[ul + vm\right]\right\} \mathrm{d}l\mathrm{d}m \quad \text{where} \quad W = \exp\left[w(n-1)\right]$$

- 3D visibility function $\mathsf{V}(u,v,w)$ can be transformed into a 3D image volume $\mathsf{B}(l,m,n)$ but non-physical as only l,m are directional cosines (i.e., 2D)
- Our lovely 2D Fourier transform now does not hold…

Wide-field imaging - 3D to 2D



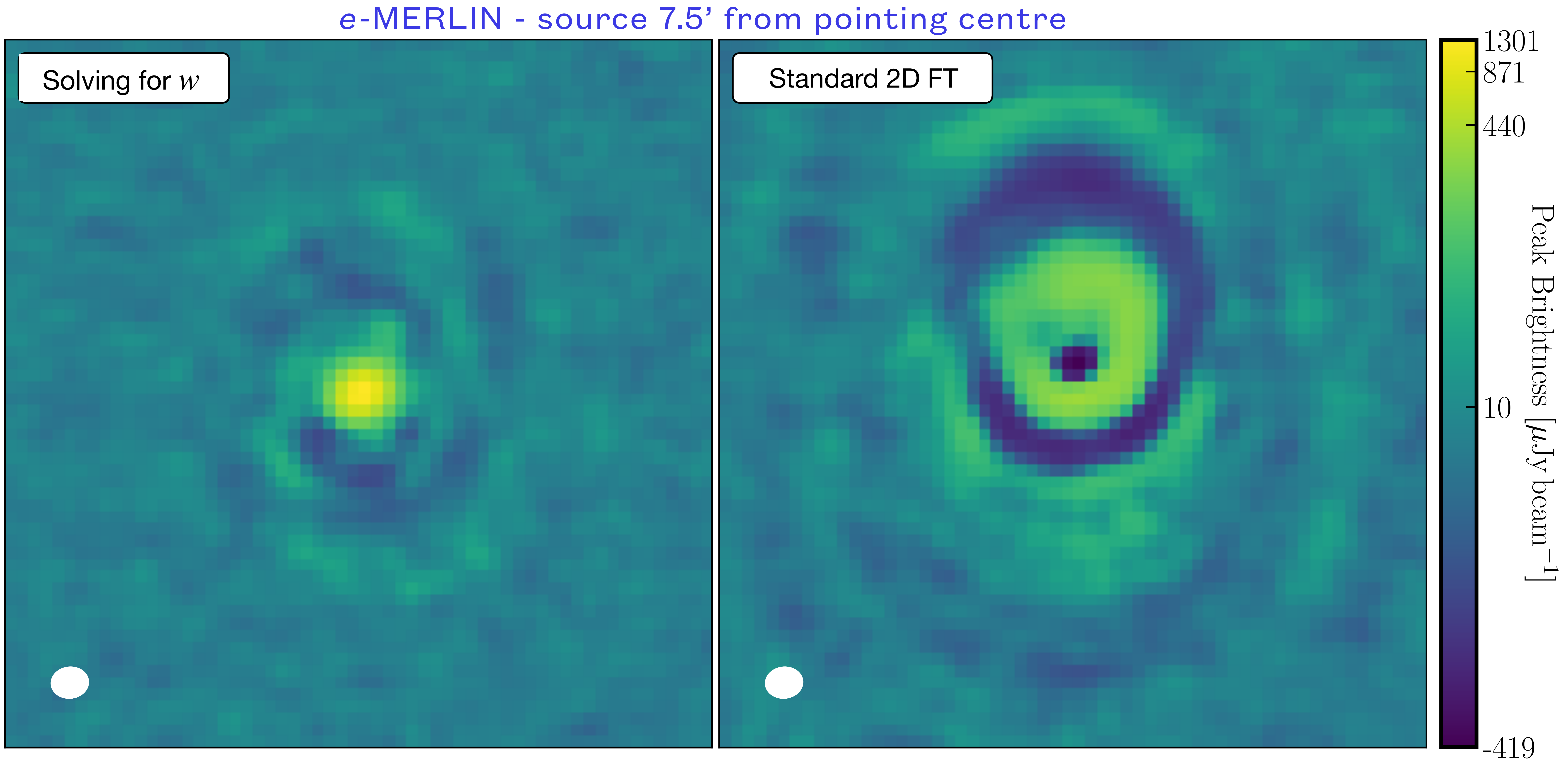
- The only non-zero values of I lie on the surface of a sphere of unit radius defined by $n = \sqrt{1 - l^2 - m^2}$
- The sky brightness consisting of a number of discrete sources ★ are transformed onto the surface of this sphere.
- The two-dimensional image ☆ is recovered by projection onto the tangent plane at the pointing centre

How do we achieve this?

1. Faceting - split field into multiple projected images and stitch together
2. Deal with the *w*-term directly (deal with the distortion when imaging)

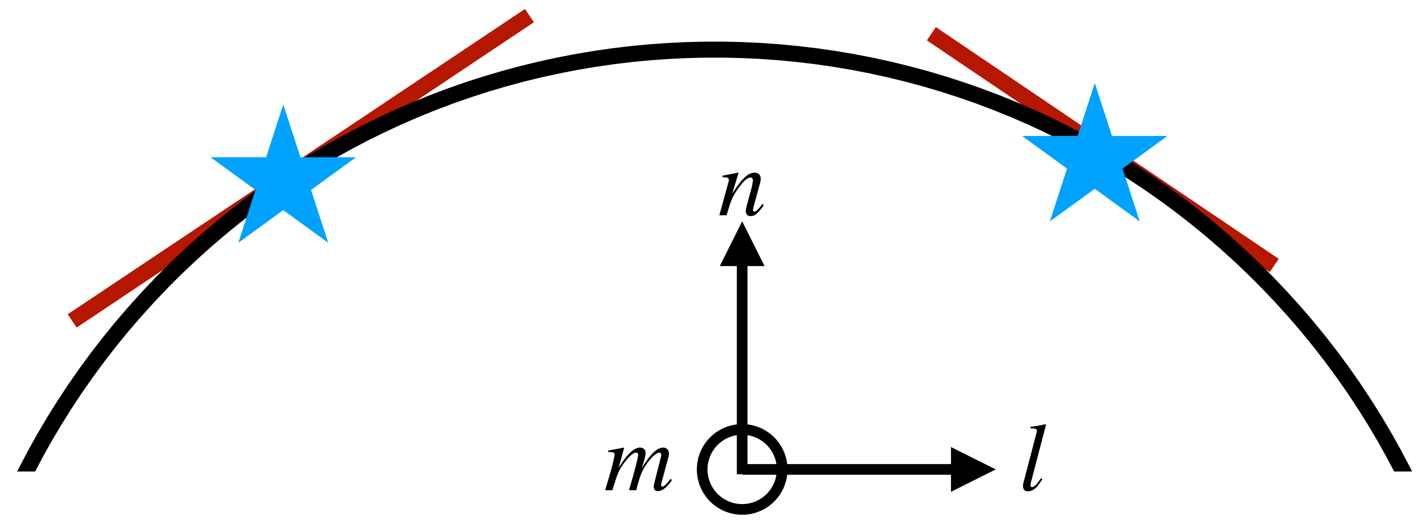
Distorting the images

- If you don't deal with the w-term:



1. Faceting

- Oldest method in the book - takes advantage of small-field approximation ($l, m \rightarrow 0$) so $W \sim 1$ so our image sphere is approximated by pieces of smaller tangent planes.
- Result → each sub-field can use the standard 2D FFT!
- Errors increase quadratically away from centre but ok if enough sub-fields are selected
- Facets can be chosen to cover known sources or overlap to complete coverage of primary beam



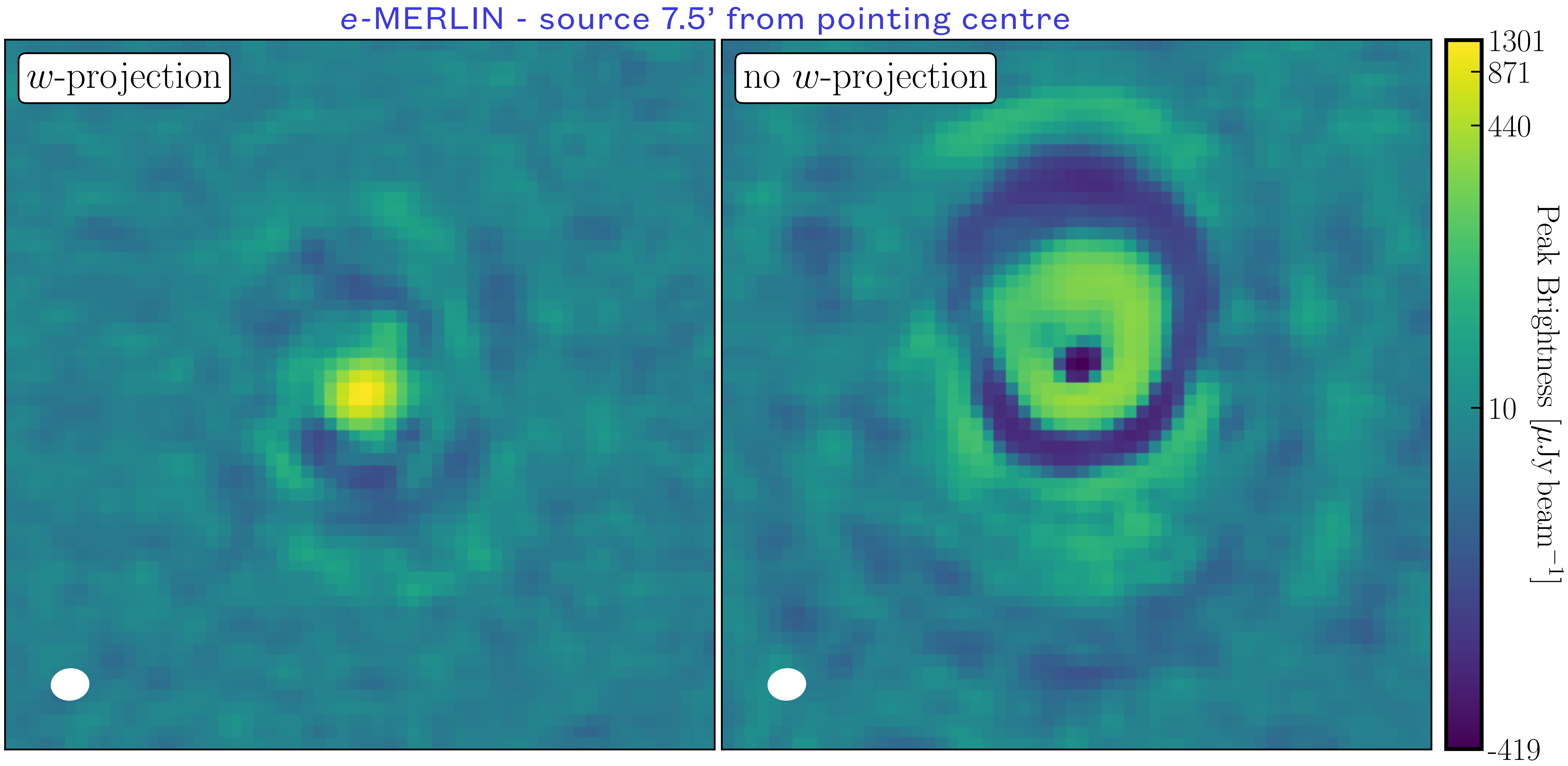
2. Dealing with *w* directly

- Other algorithms allow you to deal with the *w*-term directly when imaging (to produces a contiguous image). Examples include w-stacking and w-projection (shown next).
- To return the visibility equation to a 2D Fourier transform, the *w*-projection algorithm convolves the visibilities with the *w*-term i.e.,

$$\mathsf{V}(u, v, w = 0) * \mathfrak{F}\left(\exp\left[-2\pi i w\,(n-1)\right]\right) = \iint_{lm} \frac{\mathsf{B}(l, m)}{n} \exp\left[-2\pi i(ul + vm)\right] \mathrm{d}l\mathrm{d}m$$

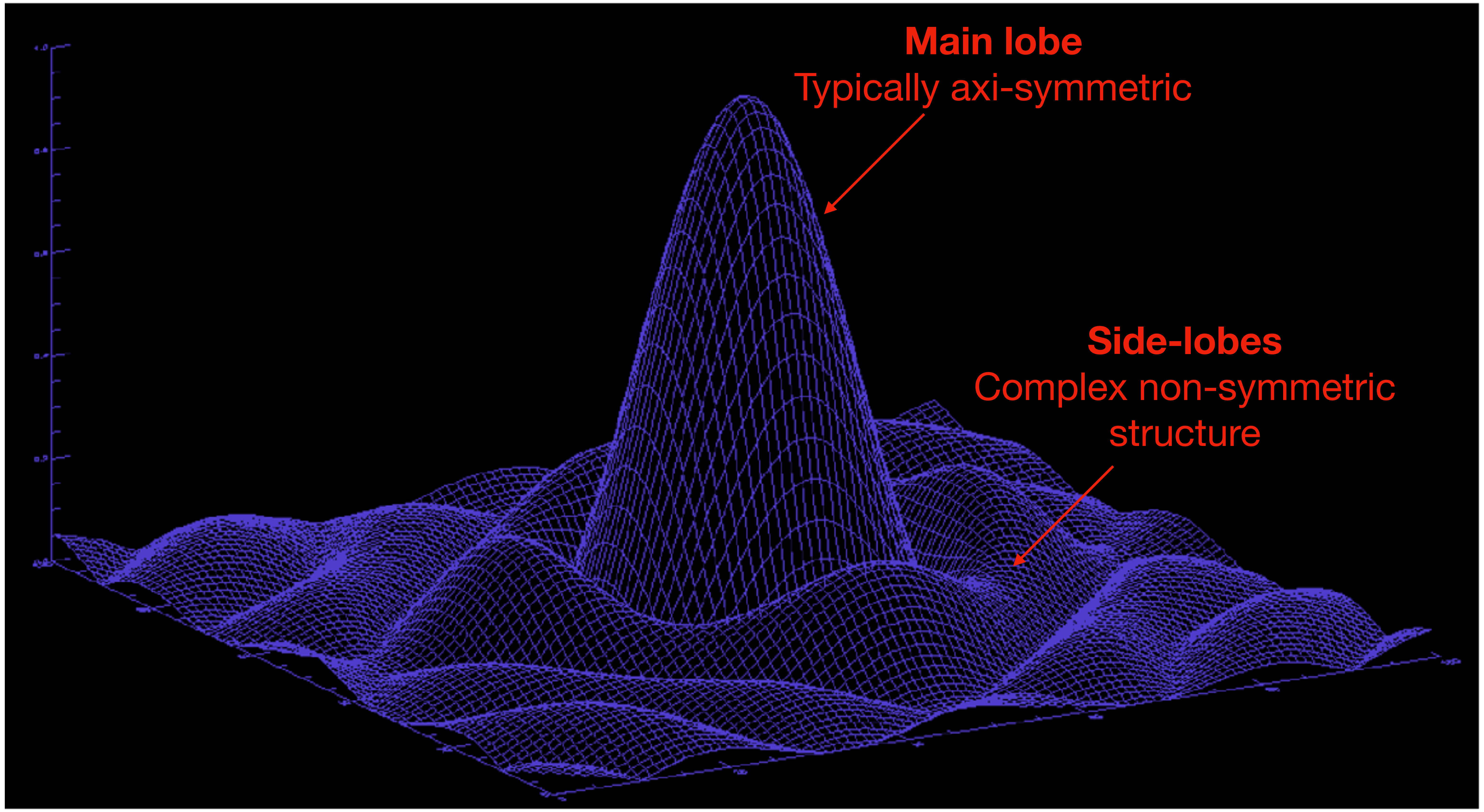
- Dependent on zenith angle, coplanarity of array and FoV.
- Deconvolution assumes constant PSF but PSF slightly changes over the image so **Cotton-Schwab algorithm** automatically used to correct for this.

Distorting the images



The primary beam

- The most ubiquitous DDE is the **primary beam response.** This is different from the PSF / synthesised beam and is related to the diffraction response of your individual antennae (i.e., λ/D not λ/B)!



Primary beam response of the e-MERLIN Knockin station

Correcting for the primary beam

- For an axisymmetric scalar/power beam A and a homogeneous array, e.g., within the main lobe:

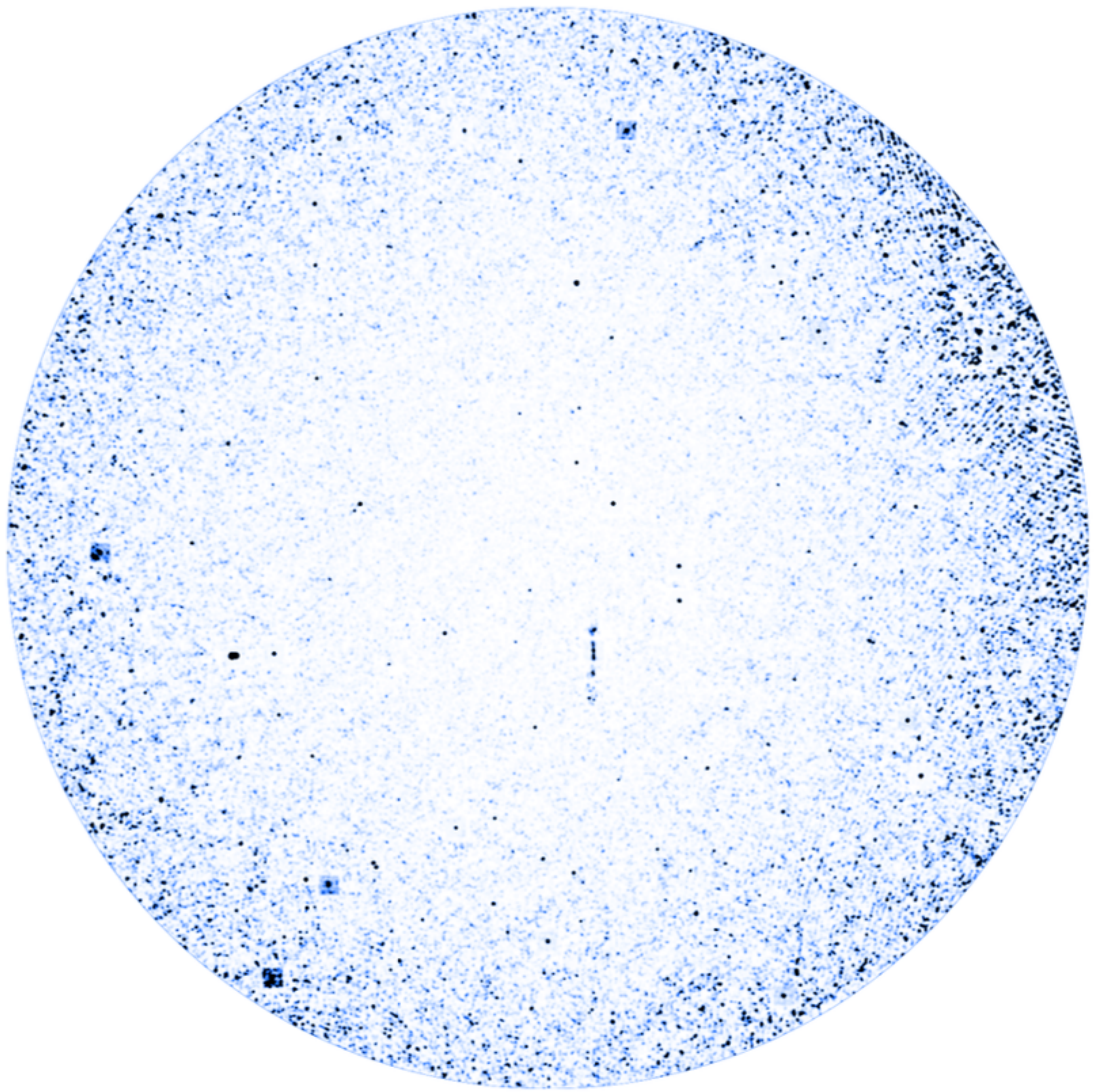
$$\mathsf{V}_{pq} = \iint_{lm} A_p K_p \mathsf{B}(l, m) K_q^H A_q^H \, \mathrm{d}l \mathrm{d}m$$

- As all A are the same on each baseline then the apparent sky is the same (hence can grid on one uv plane:

$$\mathsf{V} = \iint_{lm} |A(l, m)|^2 K_p B(l, m) K_q^H \, \mathrm{d}l \mathrm{d}m$$

- FT'ing as in imaging gives our recovered sky as $|A(l, m)^2| B(l, m)$ so we can recover the true sky by **multiplying our image by the inverse of the power beam** (voltage beam of your radio telescope squared)

Example of primary beam correction

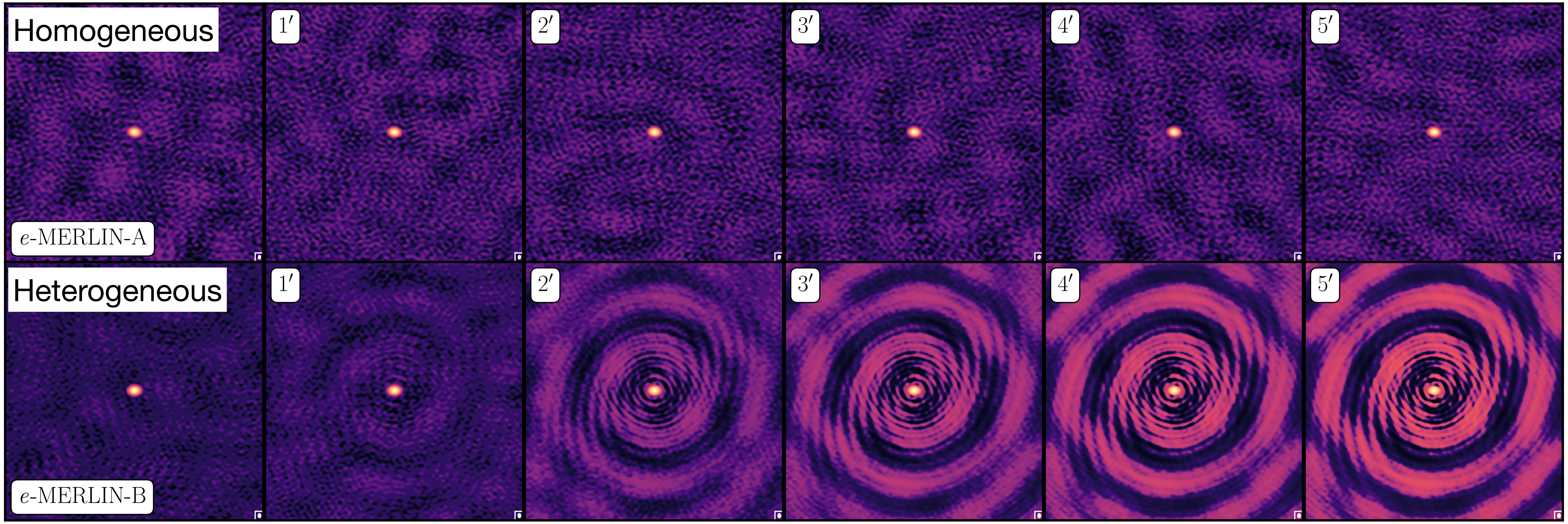


Primary beam corrected JVLA+MERLIN image of the GOODS-N deep field

Note the increased noise level towards the edge of the field

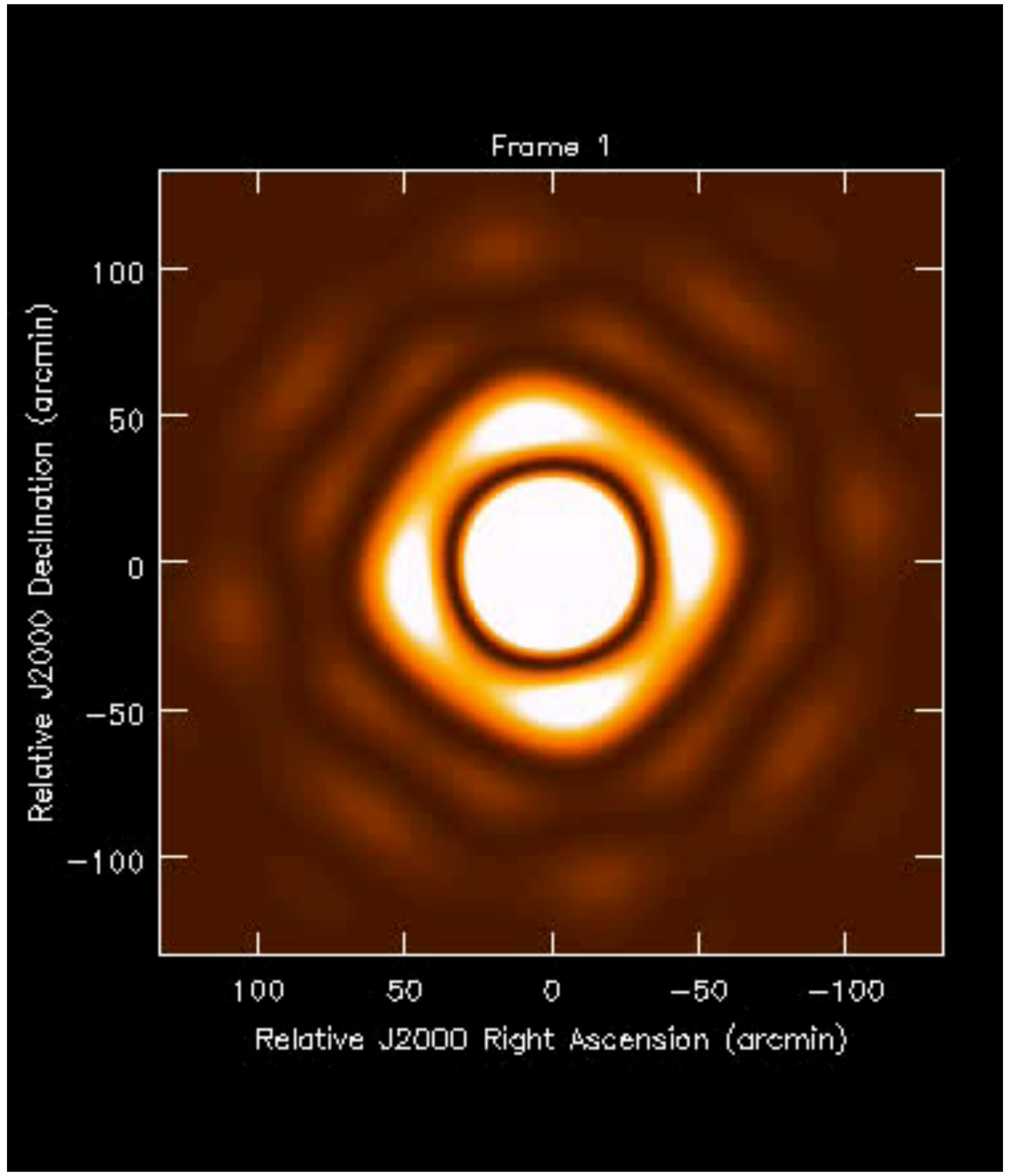
Variable and heterogeneous primary beams

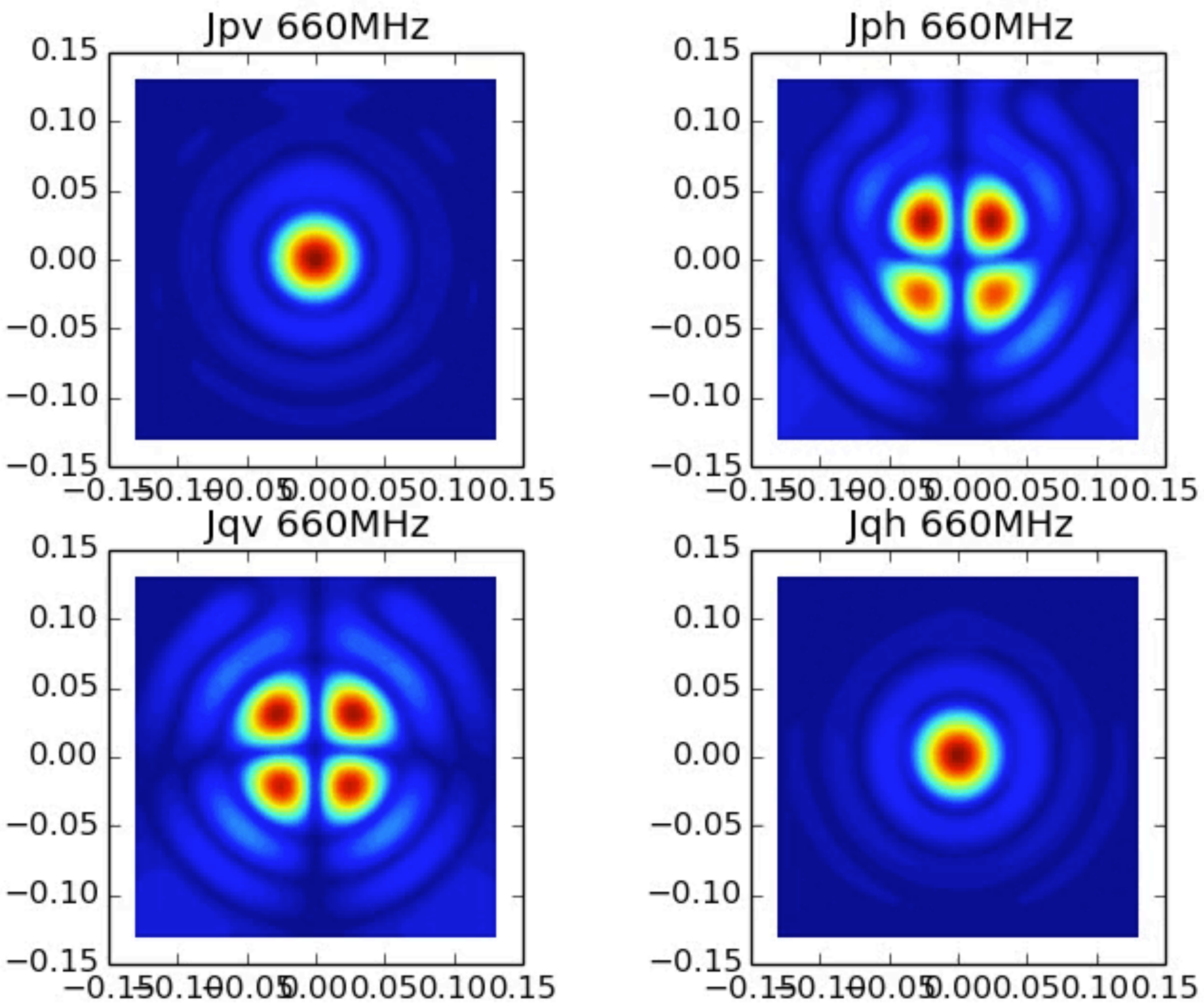
- So what happens away from the main lobe or if you have complex structure in your primary beam?
- Assumption of same effective sky $\mathrm{V}_{pq} = \iint_{lm} A_p K_p \mathrm{B}(l,m) K_q^H A_q^H \,\mathrm{d}l\mathrm{d}m$ being seen by each baseline breaks down and you get errors in your images



Homogeneous arrays also fail

- Primary beam of all arrays can vary with time and frequency!
- Has to be accounted for during cleaning and primary beam correction if imaging the whole primary beam (normally via aw-projection).





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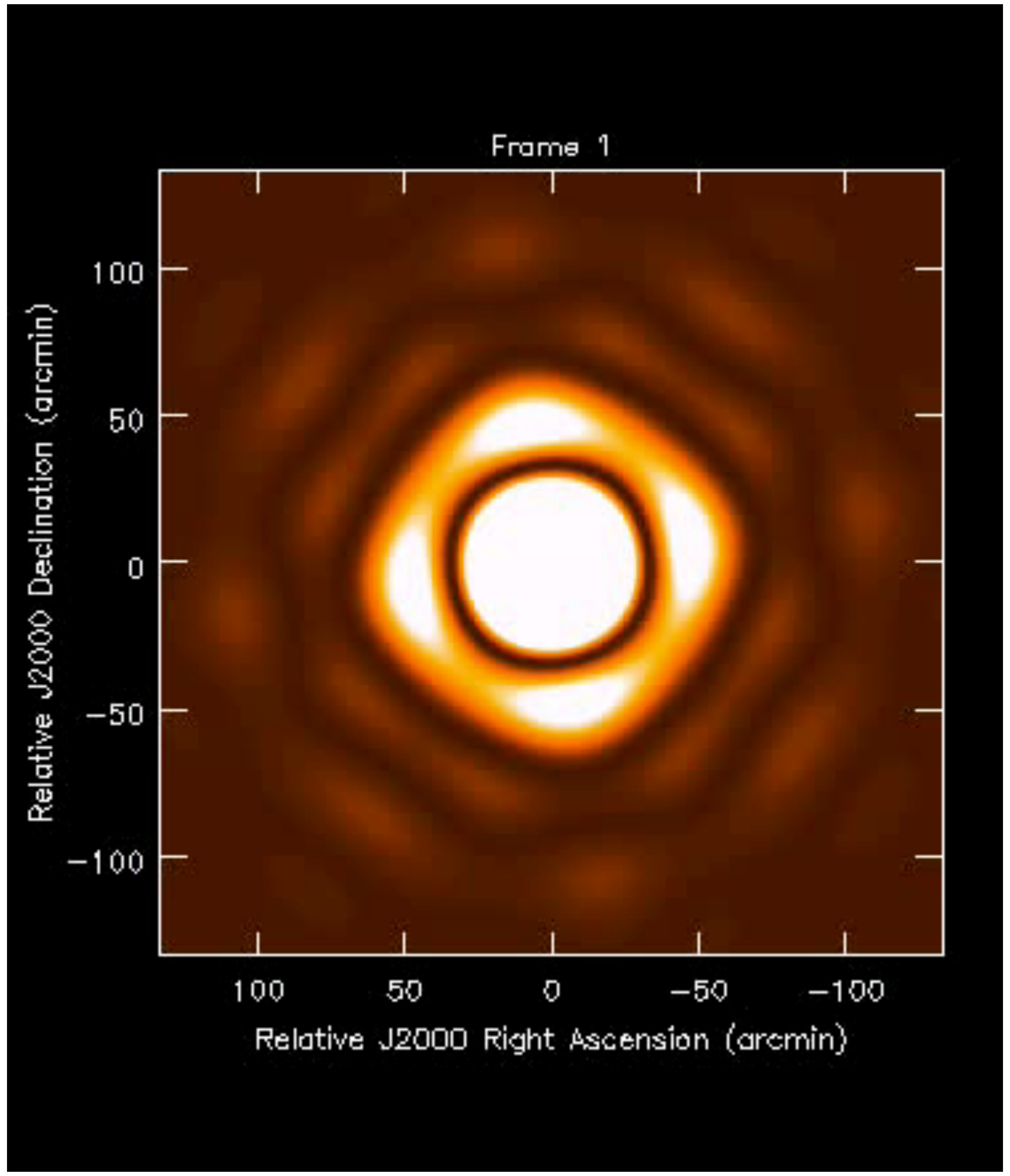

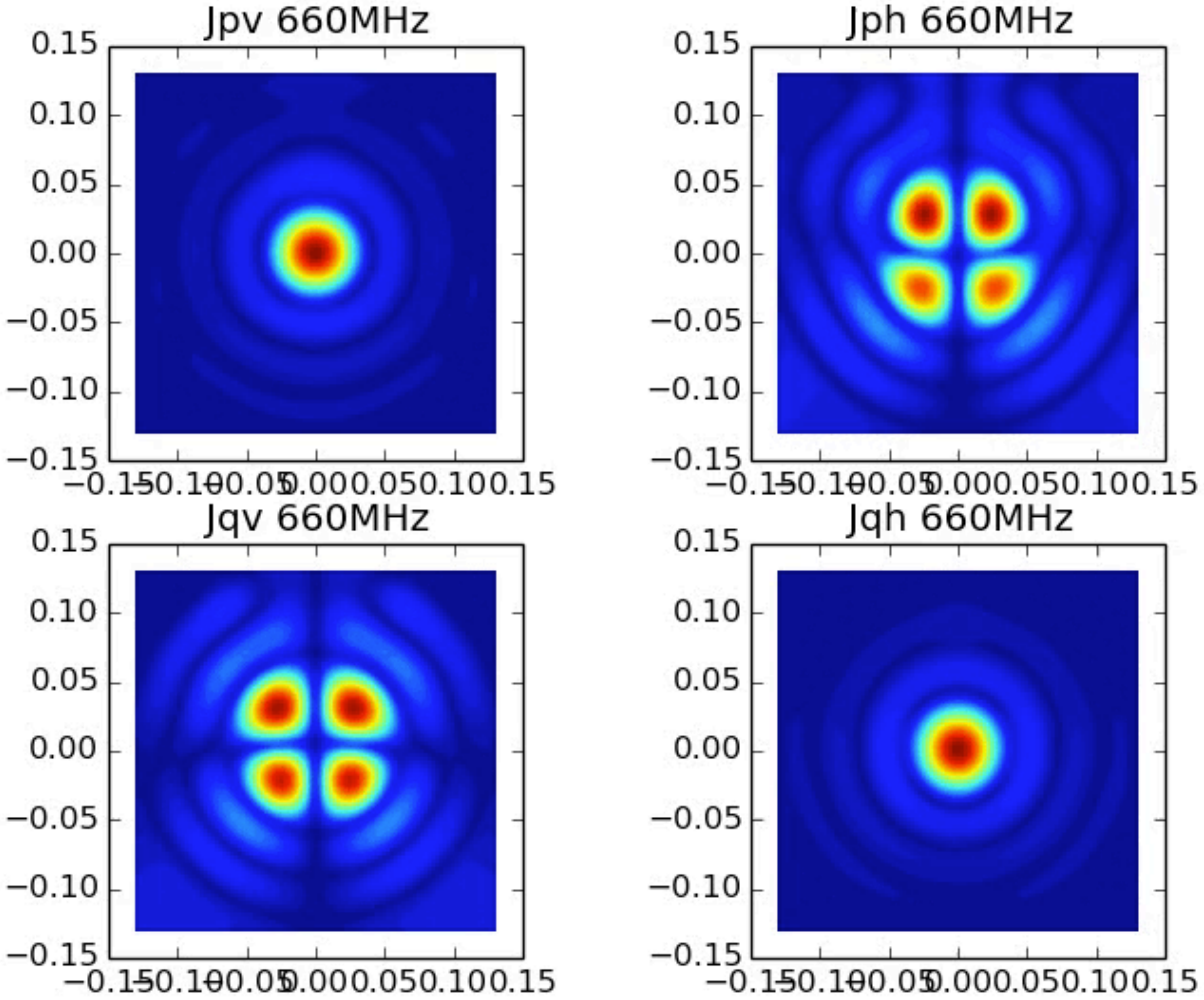

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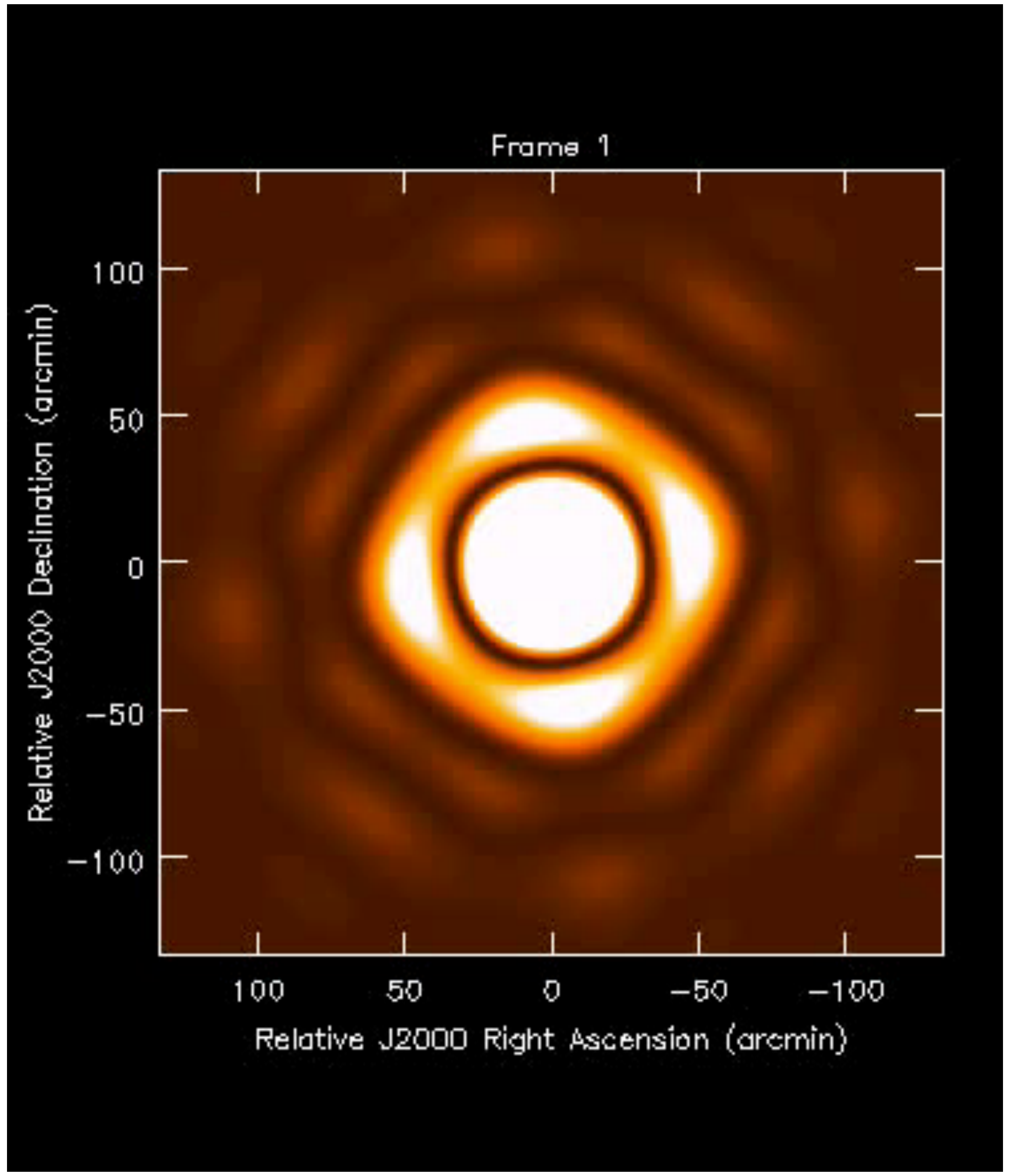

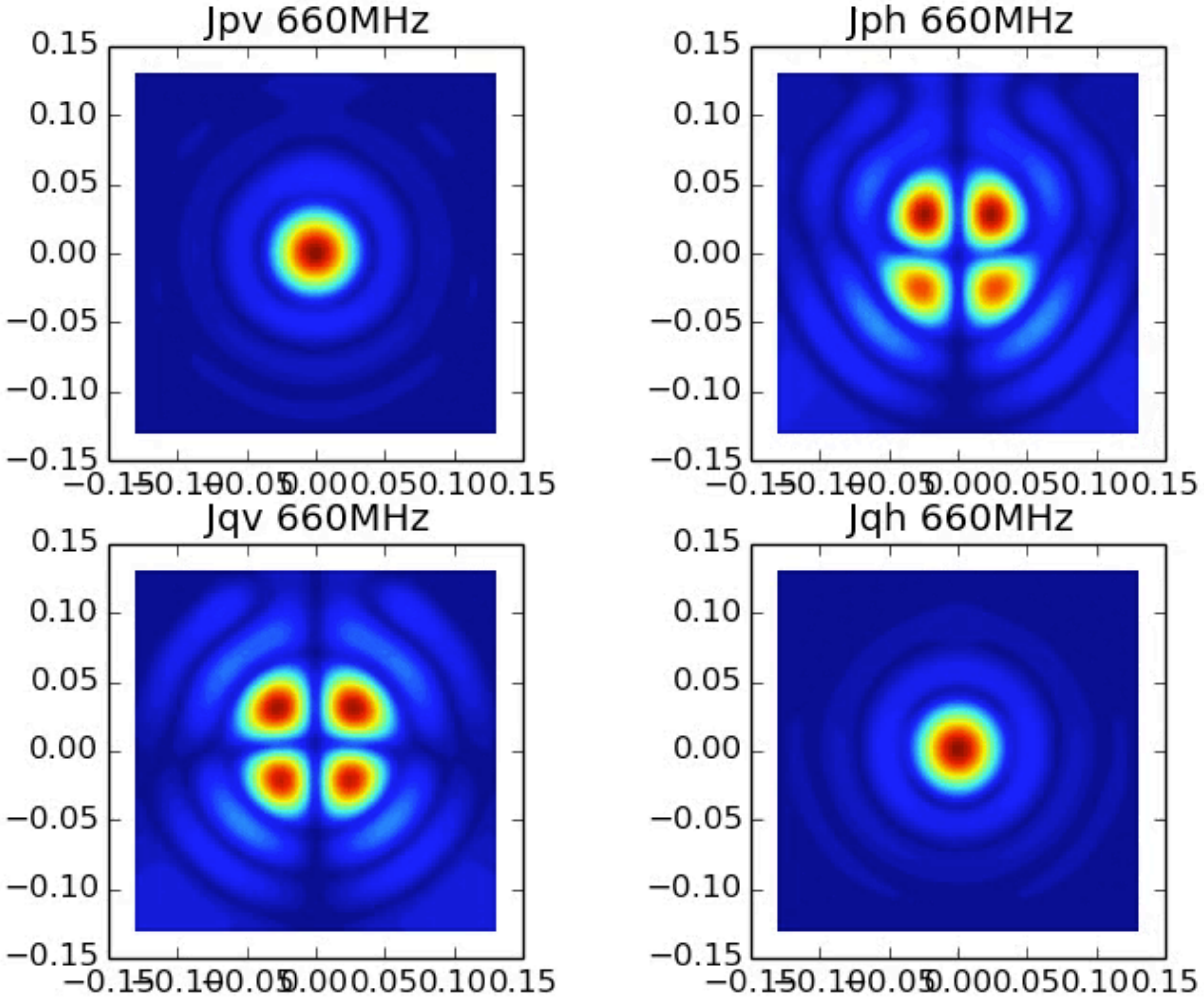

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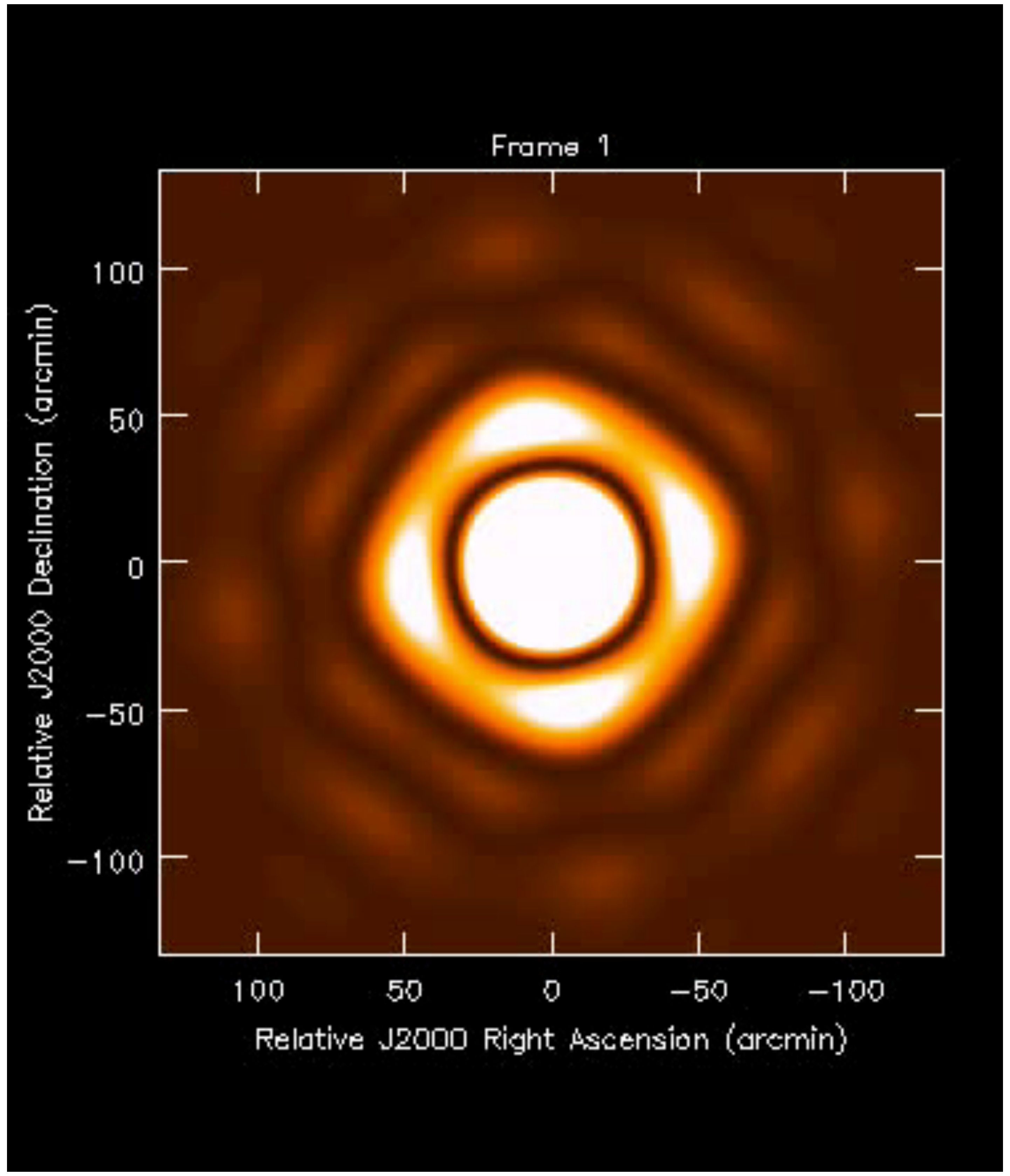
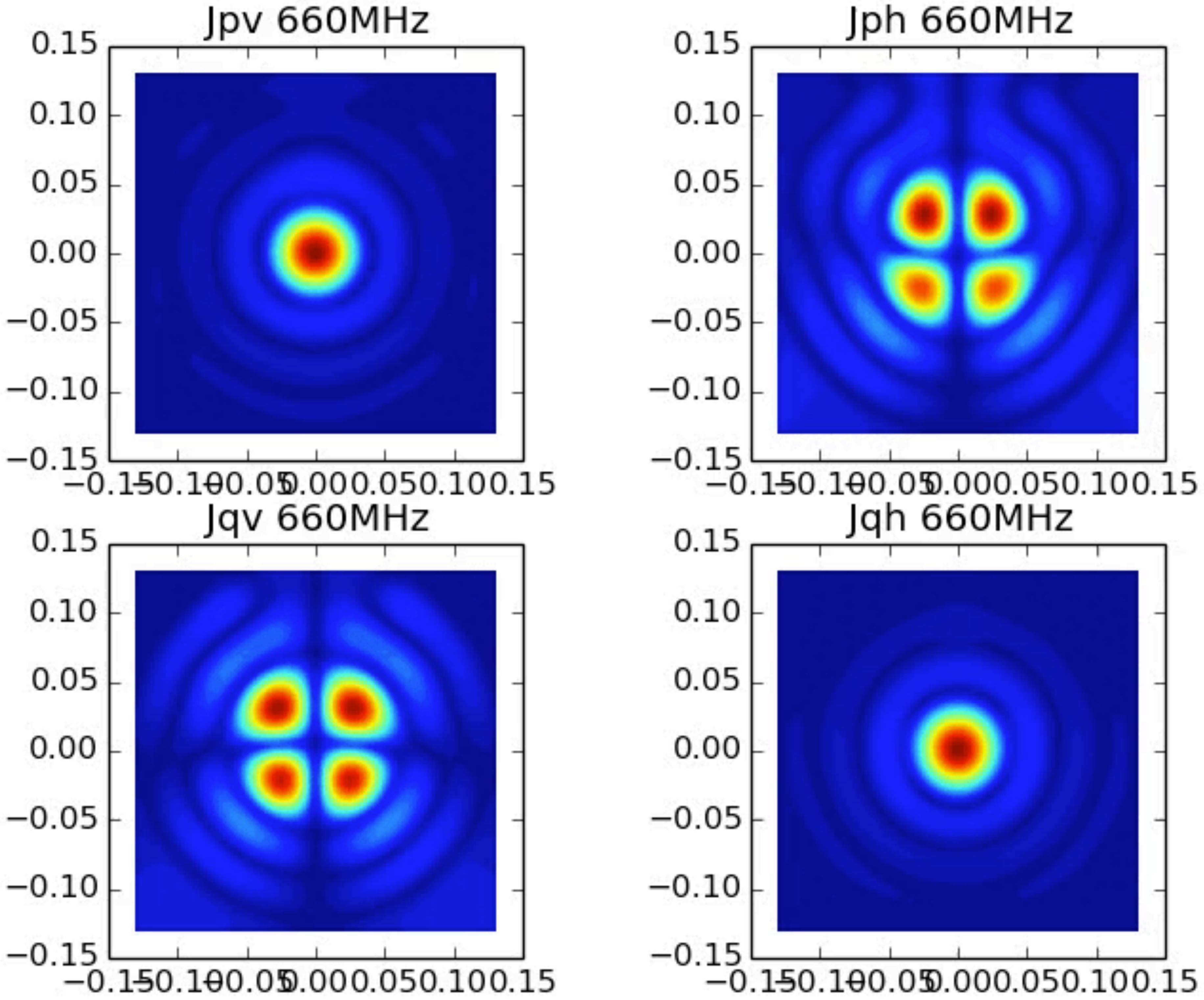
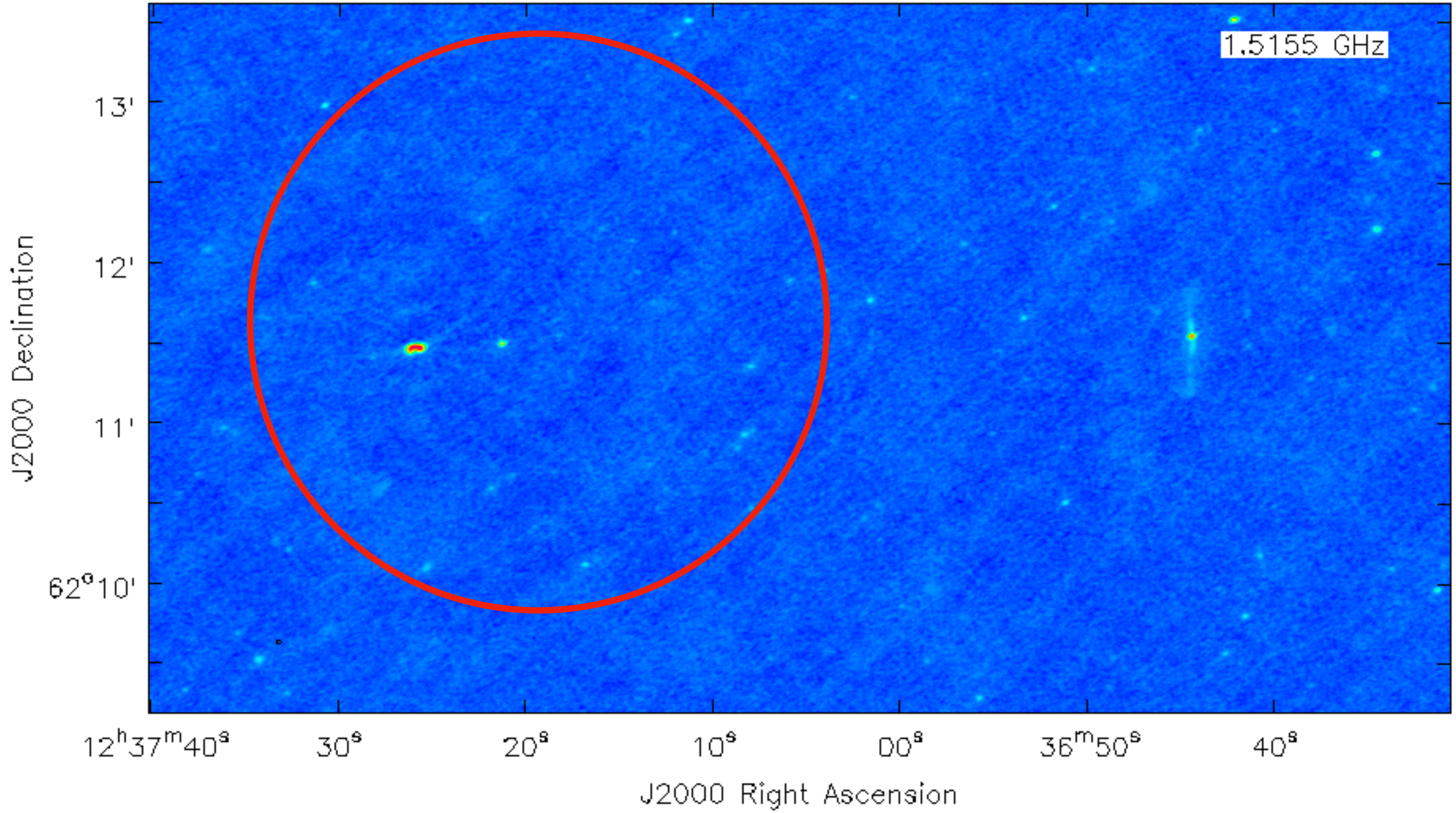




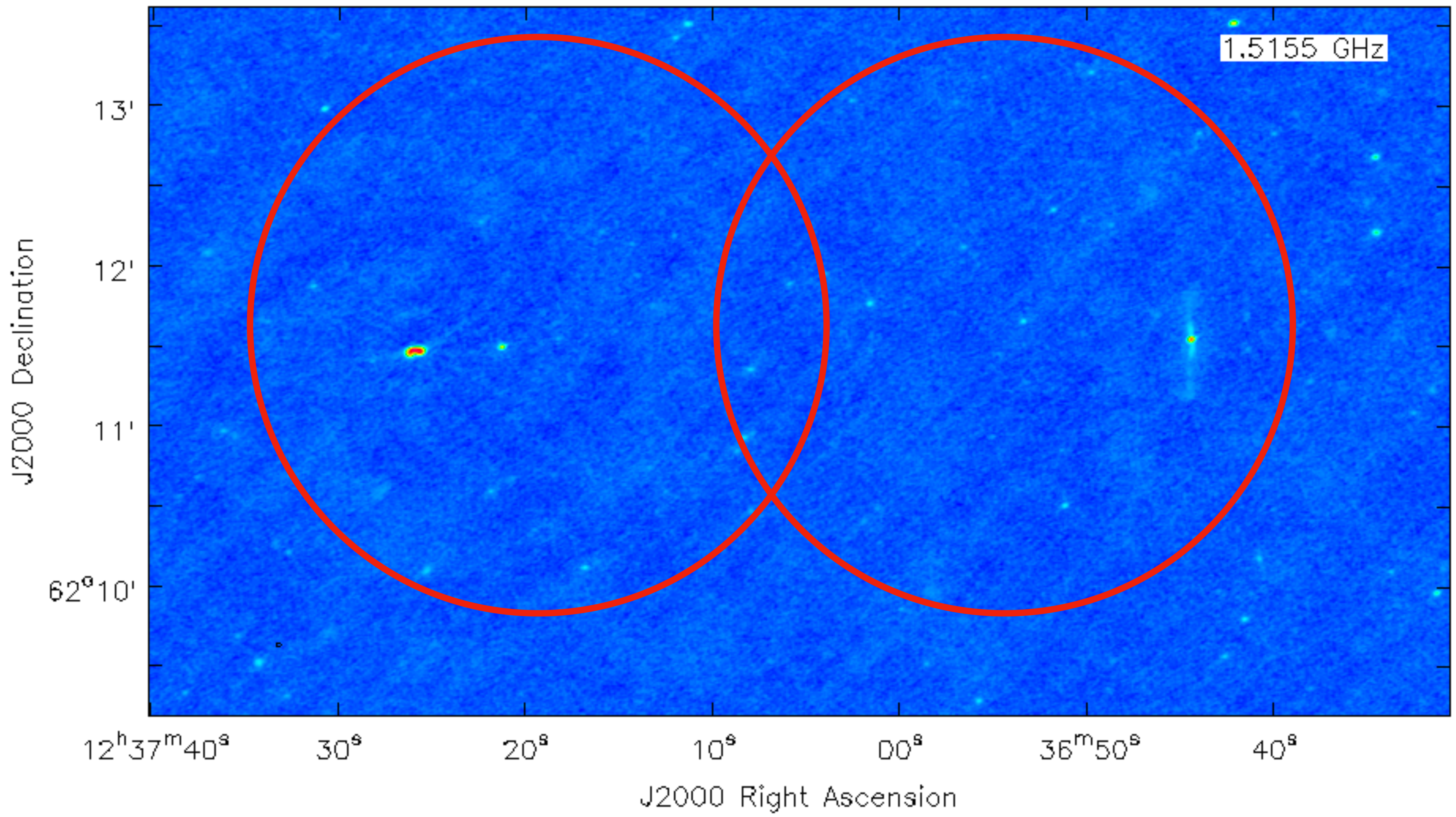
Mosaicking

What if this is our primary beam and we want to see the FR-I galaxy too?



Mosaicking

We can use multiple pointings and combine them with correct weighting



Mosaicking - the math

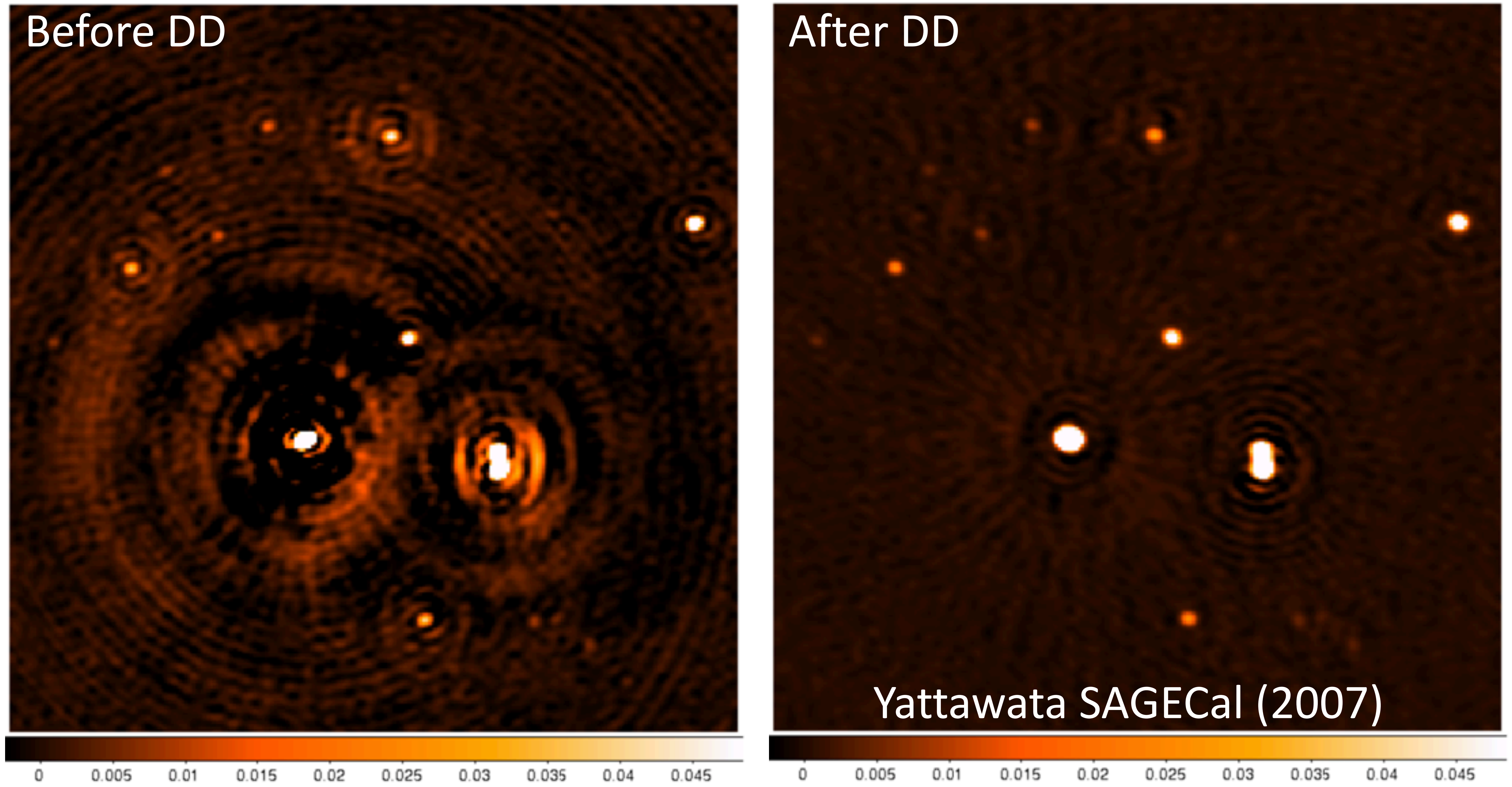
- To create the mosaicked image $\mathsf{M}(l, m)$
- Weight with $1/\sigma^2$ which is just (Primary beam)^2 or $A_i^2(l, m)$

$$\mathsf{M}(l, m) = \frac{\sum_i A_i^2(l, m)\left(\mathsf{B}_i(l, m)/A_i(l, m)\right)}{\sum_i A_i^2(l, m)}$$

$$= \frac{\sum_i A_i(l, m)\mathsf{B}_i(l, m)}{\sum_i A_i^2(l, m)}$$

Other direction-dependent calibration

- Direction dependent (DD) effects may need further corrections applied during imaging… not a fully solved problem!
- Can be ionosphere, tropospheric, instrumental (e.g. a - projection)
- Affects position, brightness & polarisation angles!

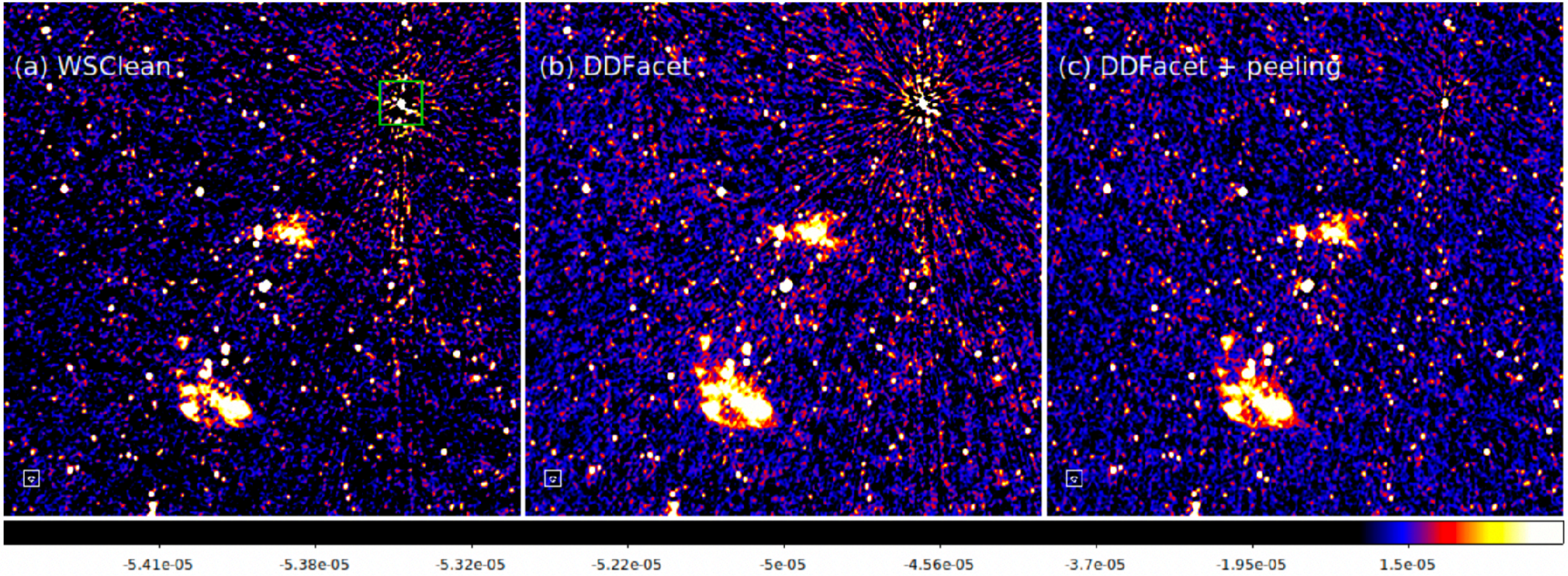


Other direction-dependent calibration

Possible solutions:

- Image in small ‘facets’ where DD's effects are constant
- Peeling / direction-dependent calibration during visibility gridding

Imaging of ZwCl 2341.1+0000
(Parekh et al. 2021)



Topics covered

- When to use multi-scale or other deconvolution methods
- The effect of and solution to *w*-terms
- Multi-term deconvolution
- Primary beam correction
- Mosaicking
- Direction-dependent effects during imaging